Blackbird

Introduction

Blackbird is a wind-powered wheeled vehicle which, it is claimed, can travel downwind faster than the wind which is causing it to move. Many people, including several respected physics professors, emphatically deny that this is possible. The designers of the vehicle have posted a video on the internet showing it in operation and accompanied it with some more or less plausible 'explanations' but naturally, this is not proof and convinces only those that are already convinced.

This article attempts to deduce the truth of the matter – but the author is not under the illusion that it will convert many.

The vehicle

The vehicle consists of a three wheeled platform on which is mounted a wind rotor whose axis is parallel to the direction of motion (and parallel to the direction of the wind)

The turbine is coupled to the wheels by a drive chain.

In the absence of any other detailed information, we shall assume that the pitch of the rotor is fixed and that the gear ratio between the rotor and the wheels is also fixed. These parameters, however, are crucial to the operation of the vehicle and must, of course, be chosen carefully at the design stage.

Basic principles of a wind rotor

In order not to become confused, certain basic principles concerning the operation of a wind rotor must be established and, once they are grasped, must be adhered to rigidly otherwise confusion will immediately set in. In addition, it is necessary to be scrupulously accurate over the signs of the quantities involved. The direction in which the vehicle is pointing is deemed to be *positive*.

First of all, some terminology and symbols.

v The velocity of the vehicle relative to the ground (in the forwards direction). This is simply the speed of the vehicle. It is, of course, always *positive* in the circumstances we are considering.

w The velocity of the wind relative to the ground (in the forwards direction). This is called the *tailwind*. It is vital to note that, in the case of a vehicle travelling *downwind*, w is *positive*.

u The velocity with which the vehicle is moving through the air (in the forwards direction) This is called the headwind. u may be positive or negative.

The first hurdle to cross is to write down the correct relation between these three quantities. To do this we note that if the headwind u is to be positive (ie if the air is moving through the rotor from front to back) then either (or both) the vehicle must be moving forward (positive v) or the wind must be moving backward (negative w). Hence we can be sure that:

$$u = v - w \tag{1}$$

The next thing to define some parameters of the rotor:

A The swept area of the rotor. This is equal to $\pi D^2/4$ where D is the diameter of the rotor.

p The pitch of the rotor is the distance which the rotor would 'screw' itself forwards if it made one revolution in some kind of jelly. p is a fixed property of the rotor. We shall assume that it is always positive.

f The frequency of rotation (in cycles per second). As with the pitch, we shall assume that f is always positive. We shall also have occasion to use the angular velocity of the rotor in radians per second. This is equal to $2\pi f$.

It is worth noting that if the rotor has a positive pitch p and rotates at a frequency f, it will 'screw' itself forwards at a speed pf.

The force exerted on (or by) a wind rotor

Now, imagine that the rotor is not geared to the wheels but is, instead connected to a motor/generator.

First consider the case where the vehicle is stationary, the blades have *positive* pitch and the wind velocity is *negative*. This is the conventional situation with a wind turbine facing into a stiff breeze. With $v = 0$ and w negative, u (the headwind) is positive. This will produce an aerodynamic force on the blades causing the rotor to rotate in the *positive* direction and the torque in the shaft will be *negative*. We do not have to consider in detail the airflow over the blades; we will simply note that the rotor has the effect of reducing the speed of the head wind, thereby extracting energy from it. If the turbine was perfectly efficient, the velocity of the air after passing through the turbine would be $-pf$. (Note that the velocity is question is *backward* hence the minus sign.)

Now suppose that the mass of air passing through the rotor every second is equal to m. Its momentum is $m \times -u$. (the headwind u is positive but the momentum of the wind is *negative*). Likewise the momentum of the air leaving the rotor is $m \times -pf$. Now, Newtons second law says that whenever a change in momentum occurs a force is exerted on the object causing the change and this force is exactly equal to the rate of change of momentum. We can, therefore, write down a very important equation relating the force F_R exerted on the rotor by a headwind. It is this:

$$F_R = -mu - -mpf \tag{2}$$

I am sorry about all the minus signs but they really are crucial. We can get rid of some of them as follows:

$$F_R = m(pf - u) \tag{3}$$

Lets check that this equation makes sense. If $f = 0$ (i.e. the blades are stationary) then $F = -mu$. This implies a backwards force on the rotor equivalent to the force of a wind on a brick wall. This is not realistic in practice but at least the force is in the right direction. In truth, the formula will only work correctly if the blades are spinning fast enough to affect all the air across the whole spinning area – a condition which we shall assume applies as soon as the vehicle is moving fast enough.

Now suppose that the rotor is made to rotate at a speed such that $pf = u$. the force exerted on it will reduce to zero. This makes perfect sense as the rotor is simply 'screwing' itself through the air.

And if pf is made greater than u (by putting power into the motor), F_R becomes positive as you would expect. This is the situation in an aeroplane. The turbine has become a propeller.

But how do we know what m is? This is a bit of a problem. You could argue that the mass of the air entering the rotor is $m = uA\rho$ where ρ is the density of air. This works for a wind turbine facing a headwind but won't work for an office fan because u is zero. You could equally well argue that $m = pfA\rho$ but this doesn't work if the blades are stationary. To get a formula that gives a realistic value for m in all circumstances, we use the *mean* value of u and pf. i.e.

$$m = \frac{(pf + u)}{2} A\rho \quad (4)$$

which gives us

$$F_R = \frac{(pf + u)}{2} A\rho(pf - u) \quad (5)$$

or

$$F_R = \tfrac{1}{2} A\rho(p^2 f^2 - u^2) \quad (6)$$

but it will only work if $(pf + u)$ is a sizeable positive quantity.

In the context of Blackbird using a tailwind, what this means that this analysis will only work when either it is travelling faster than the wind or when its rotor blades are rotating fast enough to 'claw' their way into the air. How Blackbird gets going from a standing start is another question which I will not address here.

The torque exerted on (or by) a wind rotor

The second thing we must deduce about the rotor is the torque in the shaft.

T The torque in the shaft. Torque is a twist. We shall define it in such a way that when T is *positive* the torque is trying to rotate the rotor in the positive direction. (We shall note later that at the other end of the shaft, the same torque will tend to drive the vehicle backwards.)

Consider a rotor which is stationary with respect to the ground. Since it is not moving, it can do no work. It follows that (always assuming the rotor is 100% efficient) the energy entering or leaving the rotor via the rotating shaft must be equal to the difference between the kinetic energy of the air entering and leaving the rotor. The air entering the propeller every second has kinetic energy $\frac{1}{2} m u^2$ and the air leaving it has energy $\frac{1}{2} m (pf)^2$. Now the power supplied by a shaft with torque T_R rotating at a speed f is $2\pi f T_R$. (Power = torque × angular speed). Hence we have:

$$2\pi f T_R = \tfrac{1}{2} m(p^2 f^2 - u^2) \quad (7)$$

so

$$T_R = \frac{m(p^2 f^2 - u^2)}{4\pi f} \quad (8)$$

It is important to realise that, although this argument refers to a stationary rotor, it must apply equally well to a moving rotor because it only deals with *changes* in kinetic energy – not absolutes. In the case of a propeller pf is greater than u and the torque is positive. In the case of a turbine, pf is less than u and the torque is negative.

As before, we must use the mean speed of the air to calculate m (equation (4)) from which we deduce that

$$T_R = \frac{A\rho(pf + u)(p^2 f^2 - u^2)}{8\pi f} \quad (9)$$

It is worth noting here a remarkable connection with Equation (6). The expression (p^2f^2 - u^2) appears in both formulae and f we eliminate this expression we get

$$T_R = \frac{F_R(pf + u)}{4\pi f} \qquad (10)$$

which we will find a lot more useful.

The wheels

Now we must turn our attention to the drive chain and the wheels.

We shall assume that the rotor is coupled to the wheels by a fixed gear train such that:

$$v = kf \qquad (11)$$

where k is a constant. k is, in fact, the distance the vehicle will move forward when the rotor makes one complete revolution. We shall assume that k is *positive*.

Actually, k is has another interpretation. It defines the relation between the torque in the shaft and the reaction force at the wheels which drives the vehicle forward. Suppose you attach a motor to the shaft and drive the vehicle forwards. The power put into the shaft is equal to the torque × the angular speed i.e. $2\pi fT_W$. (The reason for using a different subscript here will become clear later.) This power is being used to create a reaction force against the ground which propels the vehicle forward at a speed v . We shall call this reaction force F_W (i.e. the force exerted on the vehicle due to rotation of the wheels. As with all the other quantities, this force is defined to be *positive* when the force is *forwards* and negative when it acts against the motion of the vehicle.) The power used by a force F_W moving at a speed v is F_Wv and this must be equal to the power put into the shaft. So:

$$2\pi f T_W = F_W v \qquad \text{`(12)}$$

which, combined with equation (12) gives us

$$T_W = \frac{k}{2\pi} F_W \qquad (13)$$

The torque in the shaft

Imagine that the shaft is made of tough rubber with stripes down it. We have defined T_R in such a way that when it is *positive*, power is being transferred *from* the wheels *to* the rotor. This will give the rubber a certain direction of twist. It will also cause the wheels to exert a *drag* on the motion of the vehicle. This implies that F_W will be *negative*. This means that $T_W = - T_R$. This enables us to eliminate the torque from our equations (10) and (13) and we have:

$$\frac{F_R(pf + u)}{4\pi f} = -\frac{k}{2\pi} F_W \qquad (14)$$

or

$$F_R(pf + u) = -2kfF_W \qquad (15)$$

Clearly, one or other of F_R or F_W must be negative. In other words, if the wheels are powering the vehicle, the rotor must be exerting a drag on it; alternatively if the rotor is providing the power, the wheels must be dragging in back.

We are now, at last, in a position to bring all these equations together and find out what is going on. First we use equation (1) to eliminate u.

$$F_R(pf + v - w) = -2kfF_W \qquad (16)$$

Now equation (11) to eliminate f.

$$F_T(pv/k + v - w) = -2vF_W \qquad (17)$$

i.e.
$$F_W = -F_R \frac{pv/k + v - w}{2v} \tag{18}$$

We do the same for equation (6)

$$F_R = \tfrac{1}{2} A \rho ((pv/k)^2 - (v - w)^2) \tag{19}$$

Now what we are interested in is the total force on the vehicle so it is crucial to know which of these two forces is positive and whether or not it is larger than the negative one. To put it more simply, when the vehicle is travelling faster than the wind, does the rotor power the wheels or do the wheels power the rotor?

The surprising answer is that it is the latter. It is the wheels which power the rotor. The rotor acts as a propeller, throwing the (forward-moving) air backwards (relative to the ground) thus providing forward thrust and extracting energy from the air (which is now moving slower with respect to the ground than it was before the rotor caught up with it).

So our task now is to see if we can select positive values for *p* and *k* which will make F_R positive and $F_R > F_W$ for some speed $v > w$. Let us simplify things by writing $v/w = N$.

Equation (19) becomes:

$$F_R = \tfrac{1}{2} A \rho w^2 ((Np/k)^2 - (N - 1)^2) \tag{20}$$

This means that
$$Np/k > N - 1 \tag{21}$$
or
$$p/k > (N - 1)/N \tag{22}$$

It will be recalled that when the rotor makes one revolution, the vehicle moves forwards a distance *k*. So what this equation is saying is that if you want the vehicle to travel twice as fast as the wind ($N = 2$) then *p/*k must be at least equal to 1/2.

Now we must make sure that the magnitude of F_R is greater than that of F_W. From equation (18) we see that this condition is only satisfied if

$$\frac{pv/k + v - w}{2v} < 1 \tag{23}$$

This is crunch time. Putting $v = Nw$ we get:

$$Np/k + N - 1 < 2N \tag{24}$$
$$p/k < (N + 1)/N \tag{25}$$

Again, if we want to travel twice as fast as the wind we must make sure that $p/k < 1.5$

It is immediately obvious that putting $p = k$ satisfies both conditions. What is more, it satisfies both conditions for all values of *N* meaning that a vehicle whose propeller pitch *p* is equal to its forward ratio *k* could, in principle travel as fast as it likes!

Can it really be true?

All this messing around with equations and assumptions is all very well, but I won't really believe it until I have put some figures in the equations and actually calculated the forces on the vehicle. In addition, I want to be sure that we are not disobeying the laws of conservation of energy and momentum down the line. So lets see what happens if we put the following numbers in.

$p = 5$ m

$k = 5$ m

$w = 10\ ms^{-1}$ (about 22 mph)

$v = 15\ ms^{-1}$ (about 34 mph)

At this speed the frequency of rotation is $v/k = 3$ revs per second

The headwind $u = v - w = 5\ ms^{-1}$

The tailwind (ie the speed of the air relative to the rotor after it has passed) = $pf = 15\ ms^{-1}$

The speed of the tailwind air relative to the ground = $v - pf = 0\ ms^{-1}$. This is no accident. It is easy to show that (under ideal conditions) if $p = k$ the velocity of the tailwind air will always be zero. This proves that the vehicle is most effective when $p = k$.

The thrust produced by the rotor $F_R = ½ A ρ (15^2 - 5^2)$ (equation 6)

If we assume that the diameter of the rotor is 5 m and using the density of air (1.2 kg m^{-3}) we get $F_R = 2356$ N (over 500 lbs of thrust).

Using equation (18) we find that $F_W = (-)\ 2/3\ F_R = 1571$ N which leaves 785 N to push the car along.

This implies that the vehicle is using a power of 785 × 15 = 11,780 W (16 Hp)

Now we must make sure that the wind is losing this much power.

First we must calculate the mass of air which the rotor is using every second. The air enters the rotor at a speed $5ms^{-1}$ and leaves it t 15 ms^{-1} (relative to the rotor, of course). As we have seen, we must use the mean of these two speeds so the mass of air passing through the rotor every second is $10Ap$. which works out to be 236 kg s^{-1}.

The kinetic energy of this air before the rotor catches up with it is ½ × 236 × 10^2 = 11,780 J which matches exactly with the figure for the power output calculated above. (Remember that with $p = k$, the tailwind air has no speed at all.)

As they say – it all works out beautifully.

How slowly can the vehicle go?

The question now arises – can the vehicle go slower than the speed of the wind?

It will be recalled that the analysis above really only works if $pf + u$ is a sizeable positive quantity. Now $pf + u = pv/k + v - w$ and if $p = k$, it follows that $2v - w$ must be greater than zero. This means that the effect will only kick in when the vehicle is travelling at at least half the speed of the wind. It follows that Blackbird must have some alternative means of picking up speed from a standing start. I do not know how the builders achieve this but it explains the very curious behaviour of the machine as shown in the You-tube video. When the machine starts, the rotor blades start to rotate in the direction which is opposite to the direction in which a freely rotating fan would go. Does the pilot pedal the machine for a while? I don't know. (In a personal conversation with Rick Cavallaro, I learned that the drag of the tailwind on the vehicle itself was sufficient to get it moving.)

Could the rotor be used as a turbine to start the vehicle from scratch?

Of course – but in order to do this, the pitch of the rotor would have to be reversed.

Rather than simply making p negative, lets use p' to represent the reverse pitch of the rotor. Lets also use u' to represent the tailwind so that $u' = w - v$. We also have $u' > p'f$.

Equation (3) becomes $$F_R = m(u' - p'f) \qquad (26)$$

Equation (6) becomes $$F_R = ½ A ρ(u'^2 - p'^2 f^2) \qquad (26)$$

Similar considerations show that equation (15) will become

$$F_R(p'f + u') = 2kf F_W \qquad (26)$$

Note that both F_R and F_W are positive. The wind exerts a forward force on the rotor – and the torque in the rotor shaft enables the wheels to exert a forward force as well.

But notice also that as the vehicle picks up speed, u' decreases while $p'f$ increases and there is bound to come a point when F_R (and F_W also) become zero. This happens when:

$$w - v = p'f = p'v/k \qquad (27)$$

or

$$v = \frac{w}{1 + p'/k} \qquad (28)$$

What this means is that if the (reverse) pitch was equal to k, the vehicle could accelerate up to a theoretical maximum speed of $w/2$. If, however, the pitch could be reduced as the vehicle picks up speed, it could go faster than this – and once it was going fast enough, the pitch could be eased into the positive regime to act as a propeller. The transition would be very tricky though.

Could a swivelling rotor be used to propel the vehicle forward with the wind in any quarter?

I believe the answer is a qualified yes. Let us suppose that the vehicle is moving forward at a speed $\boldsymbol{v}$ and the wind (whose speed is $\boldsymbol{w}$) is coming from a direction at an angle θ with v. (when $\theta = 0$ the wind is a tailwind.) The headwind $\boldsymbol{u}$ must now be derived from these two velocities by means of a vector equation:

$$\boldsymbol{u} = \boldsymbol{v} - \boldsymbol{w} \qquad (29)$$

Let us suppose that the rotor is swivelled (about a vertical axis) in such a direction as always to face directly into the apparent wind.

It is easy to see that the magnitude of the headwind u is given by:

$$u^2 = v^2 + w^2 - 2vw\cos\theta \qquad (30)$$

and that the angle between the axis of the rotor and the forward direction φ will be given by:

$$\cos\phi = \frac{v - w\cos\theta}{u} \qquad (31)$$

Equation (6) remains unchanged except for the addition of a cos(φ) term because we are only interested in the forward component of the force on the rotor) i.e.

$$F_R = ½A\rho(p^2f^2 - u^2)\cos\phi \qquad (32)$$

Equation (9), however, remains completely unaltered because as far as the rotor is concerned, air is entering it from the front at a speed u and leaving it behind at a speed pf. This situation will produce the same torque in the shaft regardless of the sideways motion of the vehicle. We cannot here employ the simplification used to generate equation (10) because we must bear in mind the situation when the rotor is swivelled at right angles to the direction of motion. We can still use the fact that:

$$T_W = \frac{k}{2\pi}F_W \qquad (33)$$

and hence

$$F_W = -\frac{2\pi}{k}T_R \qquad (34)$$

in which case equation (9) becomes:

$$F_W = -\frac{A\rho(pf + u)(p^2f^2 - u^2)}{4kf} \qquad (35)$$

Entering these equations into a spreadsheet is the easiest way of determining their behaviour. It turns out that if the wind is either directly behind or directly in front, the machine can extract useful energy from the wind. This is also true up to a point when the wind is on the beam but if the angle between the wind and the direction of motion of the vehicle lies between 20° and 70°, very little useful thrust can be obtained.

© J. Oliver Linton; October 2011

jolinton@btinternet.com