Einstein's Light Box and the Uncertainty Principle

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Abstract: Heisenberg's uncertainty principle is explained using the ideas of a wave packet and the true nature of the wave/particle duality is clarified in the context of Einstein's famous light box experiment .

It is ironic that the man who took the first steps along the road to quantum theory, namely Albert Einstein with his explanation of the photoelectric effect, was increasingly dismayed at the path which the theorists under the powerful influence of Niels Bohr were taking it. Throughout the 1920's, Einstein bombarded Bohr with thought experiments designed to show that his theory, and in particular the uncertainty principle discovered by Heisenberg, was fatally flawed but in each case Bohr was able to show that the measurement that Einstein proposed was not, in fact, possible and the uncertainty principle always held. Only once did Einstein cause Bohr any real trouble and that was during the sixth Solvay conference in Brussels in 1930 when Einstein presented Bohr with the following problem: imagine a box full of light; weigh the box; open a shutter for a short time ΔT; if a photon is seen to emerge, weigh the box again; the difference between the two weights will enable you to determine the energy of the photon (using the relation $E = mc^2$) with arbitrary precision; this contradicts the uncertainty principle which asserts that the product of the uncertainty in energy and the uncertainty in the time cannot be less than $h/2\pi$.

Bohr lay awake all night trying to come up with an answer to this riddle but he had his answer before breakfast. The flaw in the argument, he said, was that weighing the box had to take place in a gravitational field but the inevitable uncertainty about the position of the box in the field meant that, according to Einstein's own theory of General Relativity, there must be an equivalent uncertainty in the time at which the measurement was made and so the uncertainty principle was upheld. Apparently, Einstein accepted his solution.

This incident has often been portrayed by popular historians as the final clash between two intellectual giants which resulted in the complete triumph of Bohr's interpretation of quantum theory and Einstein's total defeat. Now while Bohr's supporters undoubtedly hyped up the significance of the event, the truth is that Bohr's supposed solution to the riddle is at best unnecessary and possibly completely wrong; and both men failed to grasp the true significance of the uncertainty principle. To understand why we need to understand the fundamental reason why Heisenberg's uncertainty principle exists in the first place.

The Uncertainty Principle explained

The uncertainty principle is a direct consequence of the wave/particle duality which asserts that a single photon (or, indeed, any quantum particle) propagates itself through space, not as a particle, but as a short burst of waves called a wave packet. Consider the two wave packets shown in *fig.*1 and *fig*.2 which have roughly the same wavelength (and therefore energy and momentum) but different duration:

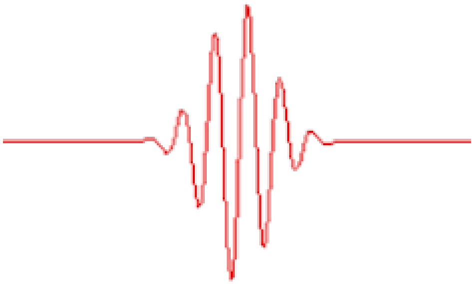

fig. 1: Short wave packet

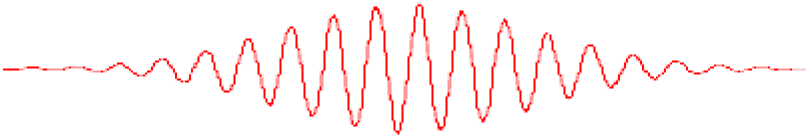

fig. 2: Long wave packet

Now lets try measuring their respective position and wavelength. The upper one contains about 5 (± ½ a wavelength) wavelengths in about 30 mm[1] so its wavelength is approximately 6mm. The *uncertainty* in the wavelength is mainly due to the uncertainty in counting the number of wavelengths in the packet and this is 10% so our measurement of wavelength is 6.0 ± 0.6 mm. Carrying out the same analysis on the longer wave packet we find that it contains 15 ± ½ wavelengths in 90 mm. The measured wavelength is therefore also 6 mm but its uncertainty is less because we have counted more wavelengths. The uncertainty is about ± 0.2 mm. (A wave packet like this does not actually have a single wavelength; it contains a range of wavelengths clustered around a single peak so in the latter case, what we are really saying is that the wave packet contains a range of wavelengths ranging from about 5.8 mm to 6.2 mm.)

Now lets consider the uncertainty in *position*. Due to the difficulty of determining where the packet begins and ends, the uncertainty in position is probably of the order of one third its length, namely 10 mm for the short one and 30 mm for the long one.

The next step is to multiply the uncertainty in wavelength ($\Delta\lambda$) by the uncertainty in position (Δx). For the short one the answer is $0.6 \times 10 = 6\ \text{mm}^2$ while for the long one it is $0.2 \times 30 = 6\ \text{mm}^2$. Not surprisingly, the answers are exactly the same because the longer the wave packet, the more accurately you can measure its wavelength – but the less accurately you can determine its position in exact proportion.

Where does the magic figure of 6 mm^2 come from? Well, suppose that the wave packet contains n waves which we can count to an accuracy of ± ½ a wavelength. It is easy to show that the uncertainty in our measurement of wavelength (λ) will be $\lambda/2n$. As for the uncertainty in position that is something of the order of $n\lambda/3$ and the product of these two uncertainties is $\lambda^2/6$. The n's cancel out. A more sophisticated argument replaces the figure 6 with 2π.

I.e. $$\Delta x \Delta\lambda = \lambda^2/2\pi \quad (1)$$

Now since the momentum $$p = h/\lambda \quad (2)$$

by differentiating (2) $$\Delta p = -\frac{h}{\lambda^2}\Delta\lambda \quad (3)$$

Now from (1) $$\Delta x = \frac{\lambda^2}{2\pi\Delta\lambda} \quad (4)$$

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[1] Note that owing to the reduction in scale when this article is printed, the actual measurements made on the printed page will differ from those I have quoted but the conclusions will be the same.

so
$$\Delta p \Delta x = (\text{-})\frac{h}{2\pi} \tag{5}$$

Alternatively we can use the other quantum relation
$$E = h\nu = hc / \lambda \tag{6}$$

hence
$$\Delta E = -\frac{hc}{\lambda^2}\Delta\lambda \tag{7}$$

Since photons move at the speed of light c, $\Delta T = \Delta x / c$ so
$$\Delta T = \frac{\lambda^2}{2\pi c \Delta\lambda} \tag{8}$$

$$\Delta E \Delta T = (\text{-})\frac{h}{2\pi} \tag{9}$$

Equations (5) and (9) represent Heisenberg's famous uncertainty principle and, as we have seen, they are a direct consequence of the wave nature of a quantum particle. Nothing else.

Einstein's Light Box Experiment

If it really is true that Heisenberg's uncertainty principle is just a consequence of the wave/particle duality, then it *must* be possible to explain why Einstein's experiment is flawed using just this concept. Gravity should not be an issue, nor should relativistic time dilation. In the first place, it would be possible to carry out the experiment in a zero gravity environment (using some kind of inertial measurement to measure the mass of the box); and second, there is no reason why we should not imagine the experiment taking place in a non-relativistic context (e.g. by using neutrons instead of photons). We have proved that, if photons have wave-like properties, it is mathematically impossible to measure both the position (or time) and the momentum (or energy) with arbitrary precision. So what really is the flaw in Einstein's argument?

Well, in a non-relativistic universe or a gravity-free environment, the uncertainty in the time at which the photon is emitted is definitely ΔT. There can be no argument about that. The shutter opens for a short time and emits a photon within that time. The wave packet which is emitted is therefore no longer than c ΔT and it contains no more than c $\Delta T / \lambda$ waves. If we were to try to measure the wavelength of this photon using a diffraction grating, there is no way that we could measure it more accurately than ± ½ a wavelength which leads to the uncertainty relation being upheld, as we have seen.

But Einstein cunningly did not suggest that we should try to measure the wavelength of the emitted photon. Instead he proposed measuring the *energy* of the photon by weighing (or measuring the mass of) the box before and after the experiment and using his relation $E = mc^2$. Einstein assumed that it would be possible to measure the masses with arbitrary position and it was this aspect of his argument that Bohr attacked. But this is not the point. Let us accept, with Einstein, that we can, in fact, measure the mass/energy of the box before the experiment, M_1, the mass/energy of the box after the experiment, M_2 and ΔT, the time the shutter was open with arbitrary precision. Einstein assumed (and Bohr tacitly agreed) that if you subtracted M_2 from M_1 you obtained the exact mass /energy of the photon at the instant that it was emitted but this is simply not true.

What Heisenberg's uncertainty principle tells us is not just that we cannot *measure* the energy of the emitted photon with arbitrary accuracy – it doesn't even *have* a precise energy when it leaves the box! You see, the wave packet pictures which I have drawn above are categorically *not* pictures of the electromagnetic waves which leave the box. When we are talking about single photons, electromagnetic waves do not exist. The wave packet is a *probability* wave whose development is determined by the photon wave equation[2] and it tells us everything there is to know about *a*) the

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[2] The 'photon wave equation' I am referring to here is a somewhat hypothetical entity which is supposed to do for

probability of finding a photon at any particular place and *b*) the range of mass/energies that it might have. When the photon leaves the box it is in a *superposition* of many states with different energies.

So if the photon has a range of energies when it leaves the box, what is the real significance of the measurement $M_2 - M_1$? Bohr knew the answer to this perfectly well. When you make a measurement on a quantum system, the system has to choose one of many different possibilities. In slightly more modern language we say that 'the wavefunction collapses' and the photon assumes one of the possible energies which are allowed it by the uncertainty principle. To take a slightly simpler example, when a photon passes through a narrow slit, its path may be bent unpredictably; and Bohr himself would have argued that as long as you refrain from looking, it is meaningless to ask where the photon is; all you can say is that it has a certain probability distribution. As soon as the photon lands on a photographic plate, however, everything changes. The angle by which the photon was deflected is now known with accuracy – but you cannot thereby infer that the angle of deflection was known when the photon left the slit. At that time, its path was *not yet determined*. It is the same with the photon and the box. At the instant the photon leaves the box its energy is not *uncertain*, it is *indeterminate*.

If Einstein and Bohr had got this far in the argument, Einstein might very well have objected at this point that if the photon only chooses to assume a certain particular value of mass/energy when it is detected by a photographic plate, how does the box 'know' how much mass to loose? This would have been a very cogent question and would have anticipated his later objections to quantum theory – that it involved 'spooky action at a distance' – by at least five years. Again, with the hindsight given to us by later experiments, we can now see that until a measurement is made, the photon and the box are in an 'entangled' state and as soon as a measurement is made on one, any measurement made subsequently on the other will have to obey the laws of conservation of energy.

In teaching A level students the rudiments of quantum theory we must avoid making two important errors. Firstly we must stop asking whether light is a particle or a wave as if these were mutually exclusive and contradictory theories. We now know the answer to this riddle. Notwithstanding the success of Maxwell's electromagnetic theory, when individual photons are detected they are definitely particles. But these particles do not fly about like molecules in a gas obeying the laws of motion; as long as they are not observed, they only exist in the form of a probability distribution. Secondly, we should stop saying that Heisenberg's uncertainty principle prevents us from simultaneously *measuring*, for example, the position and momentum of a particle with arbitrary accuracy. It is not the case that in trying to observe, say, the position of a sub-atomic particle, we cannot help disturbing its momentum. Nature is not being deliberately perverse. The truth is that a particle does not *possess* the properties of position and momentum independently before it is observed and if you constrain one, the other will take on a *range* of different possible values.

Quantum theory contains many mysteries (not least what counts as an 'observation') but the wave/particle duality and the uncertainty principle are not among their number.

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photons what Schrödinger's equation does for massy particles like electrons. I say hypothetical because no one has, as yet been able to write down such an equation which is entirely free of contradictions. When it comes to the propagation of photons, Maxwell's equations are the best we have; in describing the interactions between photons and matter, we use QED (Quantum Electrodynamics).