Newton's Principia

Simplified and rendered into modern English

DEFINITION:

THEOREM: a statement which is proved

COROLLARY: a consequence of a theorem

SCHOLIUM: an explanatory comment

LEMMA: an intermediate theorem

THE

MATHEMATICAL PRINCIPLES

OF

NATURAL PHILOSOPHY

BOOK I

OF THE MOTIONS OF BODIES

DEFINITIONS

DEFINITION I

Mass is a measure of the quantity of matter in a body and is the product of density × volume.

DEFINITION II

Momentum is a measure of the quantity of motion and is the product of mass × velocity.

DEFINITION III

Inertia is a property of a massive body which causes it either to remain at rest or continue moving in a straight line with constant speed.

DEFINITION IV

A force is an action on a body which tends to change the state of rest or uniform motion of a massive body.

DEFINITION V

A centripetal force is a force which acts towards a centre of rotation.

Examples include the force by which the planets are constrained to move in a circle and the force on a stone whirled around in a sling.

Objects moving in a circle are pulled away from their tendency to move in a straight line by a force which acts at right angles to their motion and towards the centre of rotation. A stone, projected with sufficient speed might therefore go right round the whole Earth and effectively go into orbit.

It may be that it is gravity which causes the Moon to orbit the Earth. If so, it is the responsibility of mathematicians to calculate what force may be necessary to keep it in orbit and the determine the path which an object projected from a given place with a given speed would follow.

DEFINITION VI

The absolute magnitude of a force is a measure of its strength.

[This definition does not seem to add anything new. We now measure force in Newtons – 1 N being the force required to accelerate a mass of 1 kg with an acceleration of 1 ms^{-2}.]

DEFINITION VII

The accelerative magnitude of a force is a measure of the strength of the force per unit mass.

[The accelerative magnitude of a gravitational force is F/m and is what we now call gravitational field strength g and is measured in N kg^{-1}]

Since the accelerative magnitude [or gravitational field strength] is the same at equal distances from the centre of the Earth, all objects at the same height will fall towards the centre of the Earth with the same acceleration

DEFINITION VIII

The motive magnitude of a force is a measure of the strength of the force per unit momentum.

[The motive magnitude is effectively m × g and is therefore equal to the absolute magnitude. The necessity for this definition is obscure.]

For example, the weight of a massive body may be measured by measuring the upward force needed to stop it falling.

SCHOLIUM

1. Absolute time exists independently of anything else and flows at a constant rate. Relative time is a measure of duration.
2. Absolute space exists independently of anything else. Relative space is a measure of the distance between two points in space.
3. Place is a part of space which a body takes up.
4. Absolute motion is the motion of a body from one absolute place to another; relative motion is the motion of a body from one relative place to another.

For example, if the surface of the equator has an absolute motion towards the east of 10010 units while a ship at that place moves with relative motion of 10 units towards the west on whose deck a sailor is walking with a relative motion of 1 unit towards the east, the absolute motion of the sailor will be 10001 units to the east but his motion relative to the Earth will be 9 units to the west.

[There follows a lengthy discussion concerning the philosophical difficulties of defining absolute space and absolute time.]

AXIOMS, OR LAWS OF MOTION

LAW I

Every body preserves its state of rest or uniform motion in a straight line unless acted upon by a resultant force.

Projectiles do not requires a force to keep them moving. Nor does a spinning top or the Moon or the planets.

LAW II

The rate of change of momentum of a body is proportional to the magnitude of the resultant force and is in the direction of the force.

i.e.

$$\boldsymbol{F} = \frac{dm\,\boldsymbol{v}}{dt}$$

LAW III

If a body A exerts a force on another body B, the B will exert an equal and opposite force on body A

If a horse pulls forwards on a stone, the stone will pull backwards on the horse by exactly the same amount. It follows that if as a result of an interaction between two bodies body **A** changes its momentum by a certain amount, body **B** will change its momentum by the opposite amount.

[This is effectively a statement of the conservation of momentum]

COROLLARY I

If a body is acted upon by two forces acting in different directions the single resultant force may be found by completing the parallelogram.

[Newton's proof of this theorem is not very rigorous.]

COROLLARY II

Any force may be resolved into two components in two different directions.

COROLLARY III

Provided that you count backwards motion as negative, the total momentum of an isolated system of interacting bodies remains constant.

For example, if a body A of mass 3 (units) travelling with a speed of 2 is struck from behind by a body B of mass 1 units travelling at a speed of 10, the total momentum will be $3\times 2 + 10 \times 1 = 16$. If, after the collision body A acquires an extra 5 units, B will lose 5 units of momentum. If A acquires 12 units of momentum, B will bounce backwards with -2 units of momentum.

COROLLARY IV

The common centre of gravity of two or more bodies does not alter its state of rest or motion by the actions of the bodies among themselves and is therefore either at rest or moves with uniform speed in a straight line.

For example, the behaviour of objects on board a ship is unaffected by the motion of the ship provided the ship moves with uniform speed in a straight line.

COROLLARY V

[This corollary is obscure but basically repeats that in corollary IV]

COROLLARY VI

The motions of bodies in a given space are not affected by an acceleration provided that all the bodies partake of the acceleration equally

SCHOLIUM

Law II implies that when a body is falling, it will acquire equal increments of velocity in equal times. This in turn implies that the distance fallen will be proportional to the square of the time – a fact discovered by Galileo. It also follows that the path of a projectile will be a parabola.

[Newton then describes an experiment which he appears to have actually carried out in which two pendulums of equal length but different masses are contrived to collide at the bottom. Using the apparently well-known fact that the speed of a pendulum at its lowest point is proportional to the distance between the starting point and the lowest point he verifies the law of conservation of momentum stated in corollary III. He also discusses the different behaviour of elastic and inelastic collisions and notes that when perfectly elastic balls are used. The combined velocity of approach is always equal to the combined velocity of recession. This is, of course, a consequence of the law of conservation of energy but Newton was not to know this.

With respect to Law II Newton points out that although it is the lodestone which is the active agent in attracting a piece of iron, the iron must attract the lodestone with equal force otherwise they would move off with ever increasing speed]

OF THE MOTION OF BODIES

SECTION I

[Newton proves various lemmas which basically set out his method of successive approximations to calculate the area under a curve. In effect he is laying down the foundations of the integral calculus]

SECTION II

[Newton proves a number of theorems concerning centripetal forces (i.e. force which always act towards a fixed point.]

PROPOSITION I

Any body acted upon by a force acting towards a fixed point (but not necessarily constant) will sweep out equal areas in equal times.

[This is, of course, Kepler's second law but Newton does not mention him. Newton's proof is ingenious and is independent of the nature of the centripetal force. It is, of course, a consequence of the law of conservation of angular momentum]

PROPOSITION IV

Cor. I: The centripetal force acting on a body moving in an arc is proportional to the square of the velocity and inversely proportional to the radius of the arc.

i.e $$F_{\text{centripetal}} \propto \frac{v^2}{r}$$

[A large number of theorems follow mainly concerned with determining the shape of a trajectory given certain conditions.]

PROPOSITION XXXI

To find the place of a body moving in a given elliptic trajectory at any assigned time.

[This is a difficult problem because it does not have a straightforward analytical solution. Newton gives an approximate method.]

SECTION III - VII

[These sections concern bodies moving in conic sections e.g. ellipses]

PROPOSITION XI

If a body moves in an ellipse around a fixed point it follows that the force on it obeys an inverse square law

[This was the problem which Halley and Hooke could not solve and which prompted Halley to consult Newton in 1684]

SECTION VII

Concerning the motion of a body moving in a straight line acted upon by a centripetal force

SECTION VIII

Concerning the orbital motion of a body acted upon by a centripetal force

[Newton discusses the general case of a body moving under the action of any kind of centripetal force but surprisingly he does not discuss specifically what the shape of the orbit will be under an inverse square law, possibly because he has already shown that it will be an ellipse in PROPOSITION XI.]

SECTION IX

Concerning precessional orbits

PROPOSITION XLV

[Newton shows that different centripetal forces will cause the semi-major axis of the orbit to precess in different ways. He shows that if the centripetal force obeys a power law $F \alpha r^p$, then for nearly circular orbits the semi-major axis will precess by an angle equal to 360/√(p+3) on each revolution. He gives the example of p = 1which leads to a precession angle of 180 degrees. This causes the orbit to be symmetrical.]

"… a body acted upon by this centripetal force will revolve in an immovable ellipse whose centre is the centre of force.”

[He also considers the case of p = -1 which leads to an angle of precession equal to 360/√2. and p = -11/4 which leads to a precession angle of 720 degrees]

“...therefore the body parting from the upper apsis … will arrive at the lower apsis when it has complete one entire revolution”

[He then goes on to prove that if the centripetal force obeys an inverse cube (or any higher power) law, the body will either spiral in towards the centre or escape to infinity.

Finally, he mentions the inverse square case, almost in passing]

“...If the body after each revolution returns to the same apsis and the apsis remains unmoved then … the decrease of the forces will be in duplicate ratio of the altitude...”

SECTION X

Sundry proofs concerning the motions of pendulums and other bodies in motion

SECTION XI

Concerning the motion of two or more mutually attracting bodies

SECTION XII

Concerning the attraction of spherical bodies

PROPOSITION LXX

The force acting on a body placed inside a spherical shell and attracted by an inverse square law is zero

PROPOSITION LXXI

The force acting on a body placed outside a spherical shell and attracted by an inverse square law is the same as if it was attracted to the centre of the shell

PROPOSITION LXXIII

The force acting on a body inside a solid sphere and attracted by an inverse square law is proportional to the distance from the centre of the sphere

PROPOSITION LXXIV

The force acting on a body placed outside a solid sphere and attracted by an inverse square law is the same as if it was attracted to the centre of the sphere

[This theorem is of vital importance when Newton considers the attraction of an apple by the whole Earth.]

PROPOSITION LXXV

Two spheres will attract each other with a force which is inversely proportional to the square of the distance between their centres

SECTION XIII

Concerning the attraction of non-spherical bodies

SECTION XIV

Concerning the motion of bodies passing from one attracting medium into another

PROPOSITION XCIV

When a body passes from one attracting medium into another, the sine of the angle of emergence is proportional to the sine of the angle of incidence

[This is, of course, Snell's law (also known as Descartes' law) for the refraction of light. Newton's 'proof' is highly dubious. For example, he assumes that the attractive force on the body passing from one medium into another is constant. This is a wonderful example of a theory – which later turns out to be false – being manipulated to fit the observed behaviour perfectly!]

PROPOSITION XCV

When a body passes from one attracting medium into another, the ratio of the velocity of emergence to that of incidence is equal to the ratio of the sine of the angle of incidence to the sine of the angle of emergence

[This implies that for a situation like that of light entering glass where the angle of emergence is smaller than the angle of incidence, the speed of light in glass should be greater than the speed of light in air. This was shown to be false by Foucault in 1850.]

PROPOSITION XCVI

In a case where the velocity slows down, total reflection may occur

SCHOLIUM

"These attractions bear a great resemblance to the reflexions and refractions of light … as was discovered by Snellius; .. and by Des Cartes."

[Newton goes on to suggest that the bending of light round a sharp edge is further evidence of the attraction between solid object and corpuscles of light.]

SCHOLIUM

"But the different refrangibility of different rays is the real obstacle that hinders optics from being made perfect by spherical or any other figures."

BOOK II

OF THE MOTIONS OF BODIES (cont.)

SECTION I

On the motion of bodies that are resisted in the ratio of the velocity

PROPOSITION I

If a moving body is acted upon by a resistive force which is proportional to the speed of the body the loss in speed will be proportional to the distance travelled.

[In modern terms:

$$F = -kv$$
$$a = -\frac{k}{m}v$$

hence, by integration with respect to time

$$v = v_0 - \frac{k}{m}s$$

Newton then goes on to consider the difficult question of the exact path traced by a projectile when acted upon by such a resistive force.]

PROPOSITION II

If a moving body is acted upon by a resistive force which is proportional to the speed of the body the speed will reduce by a constant factor in equal intervals of time as will the distances travelled in those intervals.

[In other words, the speed will decay exponentially.

In modern terms:

$$F = -kv$$
$$\frac{dv}{dt} = -\frac{k}{m}v$$

hence, by separating variables and integrating

$$v = v_0 e^{-\frac{k}{m}t}$$

]

PROPOSITION III

[Here Newton discusses the motion of a body tossed up in a resistive medium such as air.]

PROPOSITION IV

[Here Newton discusses the motion of a body projected horizontally in a resistive medium such as air.]

SECTION II

On the motion of bodies that are resisted in the ratio of the square of the velocity

PROPOSITIONS V - X

[Here Newton addresses the problem of describing the motion resulting from the following differential equation:

$$F = -kv^2$$
$$\frac{dv}{dt} = -\frac{k}{m}v^2$$

This equation is easy to solve using integral calculus but Newton has decided to cast all his proofs into the language of geometry and this makes these propositions very difficult to understand. The section does, however, contain one very important passage:]

LEMMA II

The rate of change of a product of two or more quantities is equal to the sum of the rates of change of each of the quantities separately multiplied by the other quantities.

[In modern terms

$$\frac{d\left(xyz...\right)}{dt} = \frac{dx}{dt}yz... + \frac{dy}{dt}xz... + \frac{dz}{dt}xy... + ...$$

By putting x = y Newtons shows that

$$\frac{d\left(x^2\right)}{dt} = 2x\frac{dx}{dt}$$

and that in general, the rate of change of a quantity of the form xn/m is given by

$$\frac{d\left(x^{n/m}\right)}{dt} = \frac{n}{m}x^{(n-m)/m}\frac{dx}{dt}$$

This is a clear a statement of the basic principle of differentiation as you can get.]

SECTION III

On the motion of bodies that are resisted partly in the ratio of the velocities and partly in the ratio of the square of the velocity

PROPOSITIONS XI – XIV

[In the SCHOLIUM at the end of this section Newton recognises that the resistance to motion of a body through a fluid arises from two causes a) the viscosity of the medium (which results in the resistance being proportional to the velocity) and b) the density of the medium (which results in the resistance being proportional to the square of the velocity)]

SECTION IV

On the circular motion of bodies in a resistive medium

PROPOSITIONS XV – XVIII

[Newton shows that a body which is acted upon by both a centripetal and a resistive force will describe an equiangular spiral.]

SECTION V

Of the density and compression of fluids and of hydrostatics

PROPOSITION XIX

If a body of fluid is compressed the pressure is equal in all directions

PROPOSITIONS XX - XXIII

[Newton then goes on to consider the pressure and density of a spherical fluid under the influence of its own gravity and other problems.]

SECTION VI

Of the motion and resistance of funependulous bodies

[Funependulous means' hanging on a rope' i.e. a pendulum]

PROPOSITION XXIV

[This is a rather confused proposition but essentially what Newton is saying is that the period of a pendulum of given length depends on both the mass and the weight of the bob. If you could increase the mass of the bob without increasing its weight (e.g. you could double the mass but take the pendulum to a smaller planet where gravity is half) the period of the pendulum would increase (by √2). Conversely, if you could double the weight, keeping the mass the same, (e.g. by taking the original to a bigger planet) the period would increase (by √2).]

Cor. 1: Therefore if the periods of two equal pendulums with different masses are equal, the quantities of matter in the two masses are as the weights.

[i.e. The fact that pendulums of equal length but with bobs of different mass oscillate with the same period implies that weight is strictly proportional to mass.]

PROPOSITIONS XXV - XXXI

[There follows several theorems concerning the behaviour of pendulums in a resisting medium.]

SECTION VII

Concerning the motion of fluids and of bodies moving through them

PROPOSITIONS XXXII - XXXIII

[Newton starts by modelling a fluid as a system of massive particles milling about at random – a clear precursor to the kinetic theory of matter.]

PROPOSITION XXXIV

The resistance to motion of a sphere moving through a fluid is only half the resistance on a cylinder of the same diameter moving along its axis.

PROPOSITIONS XXXV - XL

[Newton describes several experiments and theoretical calculations on the rate at which a fluid emerges from a small hole in the bottom of a vessel and the rate at which spherical globes of different masses and diameters fall through a fluid.]

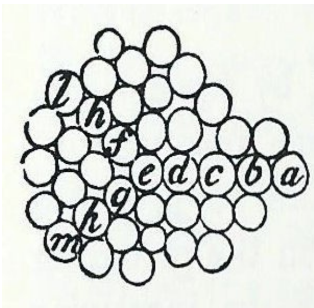
SECTION VIII

Of motion propagated through fluids

PROPOSITION XLI

A pressure is not propagated through a fluid in a straight line unless the particles of the fluid also happen to lie in a straight line.

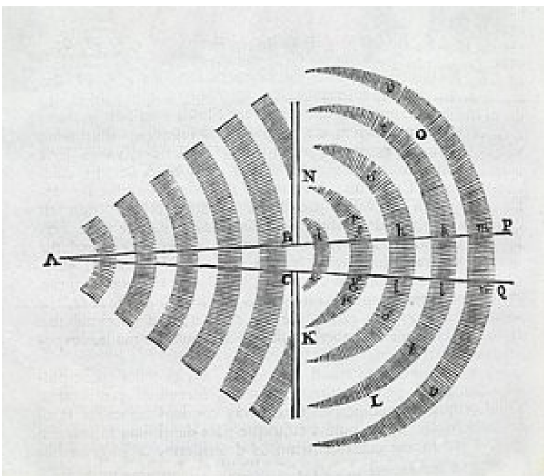
[Newton appears to imagine that a fluid consists of a, possibly random, collection of particles in contact with each other like a pile of sand and that if you push such a pile from one side, the other side will only move if the particles happen to line up exactly. He illustrates this idea with the following diagram:]



PROPOSITION XLII

All motion propagated through a fluid diverges away from the straight line.

[Newton accompanies this theorem (and the previous one) with a diagram which any schoolboy will be familiar with:



This is clearly a diagram of the diffraction of a wave through a narrow aperture. Newton obviously thinks that the the mechanism which causes a wave to spread out is closely related to the reason why pressure acts in all direction however it is applied. This is, of course, false. Nevertheless, his comments on diffraction are interesting]

Let a motion be propagated from the point A through the hole BC and let it proceed in the conical space BCQP defined by straight lines diverging from the point A... Then, because the water in the ridges of the waves is higher than in the unmoved parts of the fluid it will run down from off the tops of these ridges this way and that way …; and because the water is more depressed in the hollows than in the unmoved parts it will run down into these hollows out of the unmoved parts... That these things are so anyone may find by making the experiment in still water.

[Newton then goes on to compare this experiment with the behaviour of sound waves in air.]

… and if [sounds] come in through a window. They disperse themselves into all parts of the room and are heard in every corner.

PROPOSITION XLIII

Any vibration of an elastic medium will cause waves to radiate in all directions.

PROPOSITION XLIV

The period of oscillation of water in a U-tube is equal to the period of a pendulum whose length is equal to half the length of fluid in the tube

[This is a curious theorem which has nothing to do with either what precedes it or what follows. Newton is perfectly right, however and must have discovered the effect experimentally. He justifies the behaviour theoretically by comparing the restoring force when the system is displaced from its rest position.]

PROPOSITIONS XLV - XLVII

The velocity of a wave is proportional to the square root of its wavelength.

Newton claims that a wave will move a distance approximately equal to its wavelength in the time it takes for a pendulum of the same length to make half an oscillation.

[Although Newton's justification for this statement is very dubious, his intuition is spot on. What he is claiming is that for a wave of wavelength 1 m:

$$v_{Newton} = \frac{\lambda}{\pi\sqrt{\lambda/g}} = \sqrt{\frac{g\,\lambda}{\pi^2}} \approx 1.00\,ms^{-1}$$

The correct relationship is:

$$v_{gravity} = \sqrt{\frac{g\lambda}{2\pi}} = 1.25\,ms^{-1}$$

It is, perhaps, not too surprising that the formulae Newton came up with is pretty close to the correct one as it is the only one which is dimensionally correct.]

PROPOSITIONS XLVIII - L

The velocity of a wave in an elastic medium is proportional to the square root of the elasticity of the medium and inversely proportional to the square root of its density.

[Once again, Newton is spot on here. He goes off the rails, however, when he tries to calculate the actual value for the speed of sound. In a later SCHOLIUM Newton does the following calculation (which I have translated into modern units):

Height of mercury barometer = 0.76 m

Height of 'air' barometer = 0.76 × 13700 / 1.2 = 8677 m

Period of a pendulum of length 8677 m = 185s

Now according to a previous theorem (not discussed here), Newton claims that in this time, the sound will travel a distance equal to the circumference of a circle whose radius is 11250m. This allows him to deduce that

The speed of sound = 2 π × 8677 / 185 = 295 ms-1.

(Newton then goes on to argue that, in fact the speed will be a bit greater than this because his calculation does not allow for the 'crassitude of the solid particles of the air' leading to a revised estimate of 324 ms-1)

As before, it is interesting to compare Newton's method with the correct formula.

$$\text{Height of air barometer} = h = \frac{P_{At}}{\rho g} = \frac{102000}{1.2 \times 9.8} = 8677\,m$$

$$\text{Period of pendulum} = T = 2\,pi\sqrt{\frac{h}{g}}$$

$$\text{Speed of sound according to Newton} = \frac{2\pi h}{T} = \sqrt{gh} = \sqrt{\frac{P_{At}}{\rho}}$$

$$\text{Correct formula for the speed of sound} = \sqrt{\frac{\gamma P_{At}}{\rho}} \quad \text{where } \gamma = 1.4$$

So although the speed of sound has nothing whatsoever to do with the period of a pendulum, Newton again ends up with approximately the right formula!]

SECTION IX

Of the circular motion of fluids

PROPOSITION LI

If a solid infinitely long cylinder revolves at constant speed in an infinite fluid, the period of rotation of the fluid at a distance r from the axis of the cylinder will be proportional to r.

Cor. I: The angular velocity of the fluid will be inversely proportional to *r* and all the fluid will rotate at the same absolute speed.

[Newton's reasoning here is incorrect and his conclusions false. It is probable that he inserted this (and the subsequent two theorem's) simply to show that Descartes' vortex theory is incompatible with Kepler's laws.]

BOOK III

THE SYSTEM OF THE WORLD

RULES OF REASONING IN PHILOSOPHY

RULE I

In explaining any physical phenomenon we should make as few assumptions as possible.

RULE II

As far as possible we should try to explain similar phenomena in similar ways.

For example: respiration in man and animals; the falling of stones in Europe and America; the light of a fire and of the sun and the reflection of light in the earth and in the planets.

RULE III

If it is found to be necessary to ascribe certain bodies with certain properties, then all similar bodies should be granted the same properties, even those bodies beyond the reach of our experiments.

[Newton is clearly thinking here that if Earthly bodies like apples are affected by gravity then heavenly bodies like the Moon must also be affected by gravity.]

RULE IV

Any generalisation which is consistent with all the evidence may be regarded as verified until such time as a contrary fact is discovered.

[This is essentially Popper's rule: theories can never be proved, they can only be falsified.]

PHENOMENA, OR APPEARANCES

PHENOMENON I

The moons of Jupiter sweep out equal areas in equal times and their periods are in proportion to the sesquiplicate (3/2) power of their mean orbital radii

[In plain language, the moons of Jupiter obey Kepler's laws. Elsewhere Newton credits Kepler with the discovery of the 'sesquiplicate' power law but only in relation to the periods of the planets. Here Newton tabulates data obtained by Cassini and others to support this observation.]

PHENOMENON II

The moons of Saturn obey similar Laws

PHENOMENON III

The planets Mercury, Venus, Mars, Jupiter and Saturn orbit the Sun

[Newton deliberately omits the Earth from this list because he want to leave open, for now, the possibility that the Sun orbits the Earth as in the Tychonic system]

PHENOMENON IV

Irrespective of whether the Earth orbits the Sun or vice versa, all the planets including the Earth obey the 'sesquiplicate' law if their periods are measured against the fixed stars

This proportion, first observed by Kepler, is now received by all astronomers.

[Newton tabulates Kepler's data.]

PHENOMENON V

The five primary planets obey the above laws only if you consider their radii with respect to the Sun.

[Newton points out that, as seen from the Earth, Mars, Jupiter and Saturn exhibit retrograde motion and so the areas swept out in equal times cannot possibly be equal as required by KII]

PHENOMENON VI

A line joining the Earth to the Moon sweeps out equal areas in equal times.

This we gather from the apparent motion of the Moon compared with its apparent diameter. It is true that the motion of the moon is a little disturbed by the action of the sun, but these errors are small and may be ignored.

[The motion of the moon is complex and Newton did not have access to accurate data.]

PROPOSITIONS

PROPOSITION I

The moons of Jupiter are acted upon by a force which acts towards the centre of Jupiter and which varies inversely as the square of the distance from that point

The same is true of the moons of Saturn

PROPOSITION II

The primary planets are also acted upon by a similar force directed towards the centre of the Sun

PROPOSITION III

The Moon is acted upon by a similar force directed towards the centre of the Earth

PROPOSITION IV

The moon is held in its orbit by the very same force which accelerates a freely falling body – gravity

The mean distance from the Moon to the Earth is, according to the best current estimates 60 Earth radii. Since the Moon orbits the Earth in 27.3 days it may be shown that its centripetal acceleration is 0.00273 ms^{-2}. If the force of gravity obeys an inverse square law then we should expect the force of gravity at the surface of the Earth to be $0.00273 \times 60 \times 60 = 9.8$ ms^{-2}. This is exactly what we observe.

SCHOLIUM

If the Earth possessed multiple moons like Jupiter or Saturn and if one of those moons just grazed the surface of the Earth, it would rotate swiftly round the Earth under the influence of a centripetal force. If the force of gravity were an additional force, it would fall to Earth twice as swiftly as it is observed to do, therefore, by RULEs I and II, these forces must be the same thing.

PROPOSITION V

Likewise, the moons of Jupiter, the moons of Saturn and all the planets orbit their respective centres of attraction because of gravity

Cor. I: The attraction of a moon towards a planet (and a planet towards the Sun) is mutual

Cor. 3: Planets attract each other and disturb their orbit accordingly. Similarly the Sun disturbs the motion of the Moon and both the Sun and Moon disturb the sea.

PROPOSITION VI

The force of gravity on a body such as a moon in the vicinity of a planet is proportional to the mass of the moon and is independent of the moon's shape or constitution.

[Newton is often thought of as a great theoretician but as the following extract shows, he was not afraid to confirm his theories by doing experiments.]

It has been, now of a long time, observed by others, that [in the absence of any resistance] all sorts of heavy bodies descend to earth from equal heights in equal times and that the equality of times we may distinguish to a great accuracy by the help of pendulums. I tried the thing in gold, silver, lead, glass, sand, common salt, wood, water and wheat. I provided two wooden boxes, round and equal: I filled the one with wood and suspended an equal weight of gold … in the other. The boxes hanging by equal threads of 11 feet made a couple of pendulums perfectly equal in weight and figure, and equally receiving the resistance of the air. And placing the one by the other I observed them to play together forward and backward for a long time with equal vibrations. … By these experiments in bodies of the same weight I could manifestly have discovered a difference a difference of matter less than the thousandth part of the whole, had any such been.

[What Newton is demonstrating here is the equivalence of gravitational and inertial mass. It is true that if different materials were affected by gravity in different ways, the period of a pendulum would indeed depend on the material of which the bob was made. It is worth noting that the only reason for making the weights of the two pendulums equal is to ensure that air resistance affects each equally.]

PROPOSITION VII

The power of a body such as a planet to attract other bodies by gravity is essentially proportional to the mass of the planet.

Cor. 1: More specifically, the total force of gravity exerted by a planet on a moon is the sum of all the forces of gravity from all of it individual parts. The reason why we do not observe the force of gravity between individual objects here on Earth is that the force of gravity produced by the

Earth itself is far greater than the force of gravity between two ordinary objects.

PROPOSITION VIII

The force of gravity between two spherically symmetric objects is the same as if all the mass of both objects was concentrated at the centre of the object

[Newton knew that the whole argument about the force of gravity on the moon being the same as the force of gravity on an apple hinged crucially on the validity of this theorem. He says:]

… I was yet in doubt whether that reciprocal duplicate proportion did accurately hold or but nearly so; for it might be that the proportion which accurately enough took place in greater distances should be wide of the truth near the surface of the planet where the distances of the particles are unequal and their situations dissimilar.

[In other words, he is worried that the greater force of gravity due to particles of matter close to the surface of the Earth would not be exactly counterbalanced by the lesser force from those on the other side of the planet. The proof of this theorem is to be found in BOOK I, PROPOSITIONS LXXV and LXXVI.

Newton goes on to use this theorem and the known periods of their respective satellites to calculate the acceleration due to gravity at the surface of the Sun, Jupiter and Saturn in proportion to that of the Earth. His figures are as follows (with the accepted figure in brackets) Sun: 23.0 (28.0), Jupiter: 2.17 (2.54); Saturn: 1.22 (1.06)

He then calculates the mass of the objects (in Earth masses) as follows: Sun: 169000 (333000), Jupiter: 159 (317); Saturn: 56 (95). It is not clear why there appears to be an error of approximately 2 in these figures. Perhaps it was just a typo.

His estimates of the densities are much closer: Sun: 0.25 (0.26), Jupiter: 0.24 (0.24); Saturn: 0.17 (0.12)]

PROPOSITION IX

The force of gravity inside a spherical planet of uniform density is proportional to the radial distance from the centre

PROPOSITION X

Owing to the extreme rarity of the space through which the planets move, the orbits of the planets will persist for a very long time

HYPOTHESIS I

The centre of the system of the world is stationary

This is acknowledged by all while some contend that the Earth, others that the Sun is fixed at that centre.

[It was, of course, inconceivable to Newton that our Solar System was anything other than the centre of the Universe.]

PROPOSITION XI

The centre of the world is the common centre of gravity of the sun and all the planets

PROPOSITION XII

The sun also moves about this centre of gravity but not very far.

[Newton discusses the extent to which the Sun moves about this common centre of gravity (noting, for example, that the common centre of gravity of the Sun/Jupiter system lies just outside the surface of the Sun)]

PROPOSITION XIII

The planets move in ellipses with one focus at the centre of the sun and their radii sweep out equal areas in equal times.

[This is, of course, a statement of Keplers Ist and IInd laws..Here Newton is more concerned with the degree to which the planets might deviate from these laws by virtue of their mutual gravitational interaction. The answer is: not much.]

PROPOSITION XIV

The aphelions and nodes of the planets are fixed

[i.e. the axes of the elliptical orbits of the planets do not precess (except in so far as they may be disturbed by the gravity of other planets.]

PROPOSITIONS XV - XVI

[These refer back to previous theorems]

PROPOSITION XVII

The diurnal motion of the planets is uniform and the libration of the Moon is due to a combination of its diurnal rotation and elliptical orbit.

[The moon rotates on its axis at constant speed but, owing to the fact that its orbit is elliptical, it does not orbit round the Earth at constant speed. So, although it always presents the same face towards the Earth, it appears to wobble slightly from side to side during the course of a month.]

PROPOSITION XVIII

The daily rotation of the planets causes them to be flattened at the poles

PROPOSITION XIX

[Newton estimates that the diameter of the earth at the equator is 0.44% greater than the diameter at the poles. The accepted value is 0.34%.While his answer is acceptably close, his method of calculation seems a little dubious.]

PROPOSITION XX

The daily rotation of the Earth causes the force of gravity on a mass at the surface of the Earth to be less at the equator than at the poles

[Newton attaches a table showing the length of a 'seconds' pendulum at various latitudes and compares it with experimental data]

PROPOSITIONS XXI – XXIII

[Newton discusses the reasons for small irregularities in the orbits of the Earth, the Moon and the moons of Jupiter.]

PROPOSITION XXIV

The tides are due to the combined gravitational influence of the Moon and the Sun

[Newton argues that in open water high tide should occur about 3 hours after the lunar 'noon' (longer in a shallow estuary). Likewise, the effect of the Sun on the height of the tide will be at about 3 o'clock (day or night).

He also notes that the height of the tides will be affected by the distance between the Earth and the Moon and Sun and that since the Earth is closer to the Sun in (the northern) winter, spring tides will be higher in winter than in summer.

Moreover, since the tides will be greatest when the the gravitating body lies in the plane of the equator, tides will be higher at the equinoxes then at the solstices. This effect combines with the previous one to make the highest tides occur in January/ February and October/November.

Since the Moon's orbit is inclined to the ecliptic, it also follows that noonday tide will be higher than the night-time tide when the Moon happens to be above the ecliptic and vice versa.]

PROPOSITIONS XXV – XXXVIII

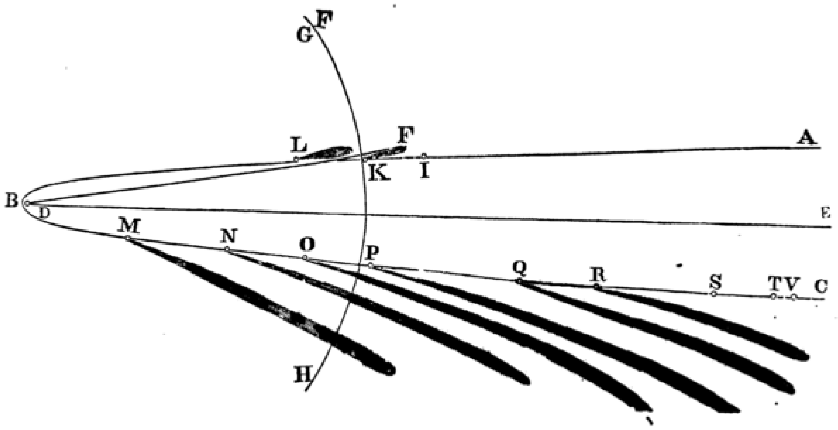
[These propositions concern the difficult problem of explaining the precise orbit of the Moon.]

PROPOSITION XXXIX

[Newton estimates the rate at which the equinoxes precess due to the combined effects of the Sun and Moon on a non-spherical Earth]

PROPOSITIONS XL – XLII

[These propositions concern the orbits of comets. Newton concludes that comets are definitely planetary bodies (i.e. beyond the Moon) and that they orbit in highly eccentric elliptical of hyperbolic orbits. He included a detailed description of the observations made on the Great Comet of 1680 which had an almost perfectly parabolic orbit and a huge tail as it passed very close to the Sun.]



<u>GENERAL SCHOLIUM</u>

[Newton concludes his book with some general comments. He starts by pointing out how one simple principle explains the motions of all the planets, their moons and other bodies such as comets. He then goes on to say... "This most beautiful system of the sun, planets and comets could only proceed from the counsel and dominion of an intelligent and powerful Being." from which he derives much conventional theology as is appropriate to his times.

Then follows this often-quoted passage:

Hitherto we have explained the phenomena of the heavens and of our sea by the power of gravity, but have not yet assigned the cause of this power. This is certain, that it must proceed from a cause that penetrates to the very centres of the sun and the planets without suffering the least diminution of its force; that operates not according to the quantity of the surfaces of the particles on which it acts … but according to the quantity of solid matter which they contain and propagates its virtue on all sides to immense distances decreasing always in duplicate proportion of the distances … But hitherto I have not been able to discover the cause of those properties of gravity from phenomena, and I frame no hypotheses; for whatever is not deduced from the phenomena is to be called an hypothesis; and hypotheses whether metaphysical or physical, whether of occult properties or mechanical, have no place in experimental philosophy. In this philosophy particular propositions are inferred from the phenomena and afterwards rendered general by induction … It is enough that gravity does really exist and acts according to the laws which we have explained and abundantly serves to account for all the motions of the celestial bodies and of our sea.

Newton concludes with a few tantalisingly short comments in which he puts forward a vision in which he imagines that the phenomena of the coherence of solid bodies, electrical attraction and repulsion, the reflection and refraction of light and even the operation of nerves in conveying sensation to the brain and causing muscles to move may one day be described by similar laws.]