The Pole in the Barn Paradox

Arthur and Betty were having another of their arguments about Special Relativity.

A: OK. I just about get time dilation and the twins paradox – after all, if you travelled directly from London to Leeds by car, your odometer would read about 200 miles, but if I did the same journey via Birmingham, it would be surprising, to say the least, if my odometer read the same as yours.

B: Yes, indeed, except that your odometer would read *more* than mine but in the case of the twins, the one who does the 'extra distance' (through space-time) ends up being *younger* than the stay-at-home twin.

A: Yes, but that's because 'distances' through space time are measured with a minus sign in front of the $(ct)^2$ term isn't it?

B: Absolutely! I am impressed!

A: Thanks. So moving clocks go slow. Yes, that makes some sort of sense. I know that it is not that the motion itself causes the clock to slow down in some sort of physical sense – because, to you, it is *my* clocks which are going slow; its just an illusion isn't it?

B: Well, not really. You can't just write of the effects of Special Relativity as some kind of illusion. It is not the case that your clocks just *look as if* they are going slowly – they *really are* going slowly. And, of course, to you, my clocks *really are* are going slow. But there is no way we can put the clocks side-by-side, as it were to compare them directly because while we can reset them to zero as they fly past each other, by the time we want to compare them a minute or a day later they are miles apart.

A: I guess so. But that has given me an idea. OK we can't put two clocks side by side but we can put two poles side by side and compare them as they go by. But according to you, my pole should be shorter than yours – but your pole should be shorter than mine! How can that be?

B: Well, it's just the same as the clocks.

A: No it isn't just the same. Look. You know that old pole of my uncle's[1] that is hanging outside the barn in farmer Jarvis' field. We can't get it into the barn because the pole is 5m long while the barn is only 4m long. But if I were to run through the barn carrying the pole at,say, 80% of the speed of light (!) according to you the pole would shrink and it might even fit into the barn!

B: You are quite right. Yes it would. At a speed of 0.8*c* the pole will be shrunk by a factor of $1/\sqrt{1 - 0.8^2} = 1.67$ so the pole will be exactly 3m long and will easily fit inside the barn.

A: But that's ridiculous!

B: No it isn't. Because you are going so fast, the pole is contracted and now it will fit into the barn – according to me at any rate.

A: Well either the pole fits into the barn or it doesn't. Surely there can be no argument about that.

B: Well...

A: OK put it like this. Suppose that at the instant the front of the pole hits the back wall of the barn, you slam the doors shut. Now either you succeed in shutting the door or you don't. If you are right, the door will shut but if I am right, at least a metre of pole will be sticking out of the barn.

B: No, No...

1 Arthur's uncle was a contender in the 1956 pole-vaulting championships in Melbourne

A: Ha! Got you! The situation is even worse than I thought! From my point of view running alongside the pole, it is not the pole which has shrunk – it is the barn! The pole is still 5m long but, to me, the barn is only 4 × 0.6 = 2.4m so over half the pole will still be outside the barn!

B: Yes, but you are forgetting that not only does special relativity affect poles and clocks, it also affects what different observers regard as simultaneous.

A: Yes, I know that – but you can't have a situation where one observer sees an intact pole inside a barn while another sees a broken pole, half inside and half outside.

B: True.

A: So which is it? Who is right? You or me?

B: I am, of course.

A: Well explain it to me then.

B: The flaw in your argument is to be found in the phrase '*at the instant the front of the pole hits the back of the barn.*' Suppose I rig up an electronic sensor which detects when the pole hits the wall. This sensor sends a signal to the door as fast as possible (i.e. at the speed of light) to cause it to shut. Lets consider the situation now from your point of view:

You are stationary, holding a pole which is 5m long. Racing towards you is a barn which is contracted to 2.4m. At the instant the front of your pole crashes through the back wall of the barn, 2.6m of pole has yet to enter the barn and at the same time a flash of light (the signal from my sensor) starts to travel towards the door – but because the signal cannot travel faster than light, and because the barn is already travelling at 80% of the speed of light, the signal can only overtake the barn at a relative speed of 20 %. This means that in the time it take the signal to travel the length of the barn (2.4m according to you) the barn itself has travelled 5 times further i.e.12m so, by the time the signal reaches the door, the back of your pole is well inside the barn.

A: Yes but that doesn't really answer my question because by the time the door slams shut, not only has the back of the pole passed through the door, it has actually passed through the whole barn! At no time is the pole wholly inside the barn which is what you seem to be maintaining.

B: I agree that, to you, there is no instant when the pole is wholly inside the barn – but to me, there is. Actually, I haven't really done justice to my argument because with the arrangement I have suggested, the door slams shut long after the instant which we are interested in. Let me suggest an alternative scenario. Suppose I put my sensor on the door itself but *delay the shutting of the door* by exactly that time which I calculate it will take for the front of the pole to travel the length of the barn. (Since the barn is 4m long and the pole is travelling at $0.8c$, this will be $5/c$ s)

A: OK lets calculate where the pole will be after this time according to me. The barn is travelling towards me at $0.8c$ and I see your clock start to tick at the instant the barn reaches the front of the pole. $5/c$ seconds later, the barn has moved $0.8c \times 5/c$ metres which is.. let me see... 4m! That still leaves 1m sticking out of the barn like I said all along!

B: Hang on a minute – you have forgotten one vital factor!

A: What's that?

B: Time dilation. To you. My clocks are going slow. Remember?

A: Oh, yes.

B: In fact every second recorded by my clock is 1.67 of your seconds so the barn will have moved that much more than 4m – 6.67m in fact; more than the length of your pole.

A: Yes, I had forgotten that. But I am still not satisfied. It is all very well proving that by the time you get round to shutting the door, the back of the pole is inside the barn, that still doesn't prove that *the whole of the pole is inside the barn*. To me the front of the pole is sticking out of the other end!

B: Yes, there is no way round it. To me, the whole of the pole is inside the barn; to you, the pole is sticking out of one end or the other.

A: Hang on. I have thought of something else. Suppose that at the instant the front of the pole hits the end wall the pole stops dead. According to you there will be no problem in shutting the door whichever sensor you use. What is going to happen now? Will the pole suddenly expand to its original length and puncture its way through the ends of the barn? And what would this look like from my point of view?

B: That's a good question. What exactly will happen? The first thing you have got to realise is that nothing, *nothing*, can travel faster than light. When the front end of the pole hits the end wall, there is no way that the back end of the pole can know about this event so it must carry on moving forwards as if nothing has changed. So the pole cannot 'stop dead' as you put it. What happens is that a shock wave travels down the pole at a speed which is, of course, less than the speed of light and by the time this shock wave has reached the back of the pole, the pole is completely inside the barn.

A: So what actually happens to the pole?

B: As the shock wave travels along the pole it physically destroys the pole and at the end of the day what we both see is the shattered remains of the pole wholly inside the barn.

A: Yes, I see. Sort of anyway. But lets go back to my original point – are the effects of special relativity just illusions or are they real? I still maintain that they are just illusions caused by the finite speed of light. I see that, to you, the pole is shorter than the barn – but we both know that the pole is *really* longer than the barn.

B: Yes we both agree that *when the pole hangs on the side of the barn* the pole is the longer but when the pole and the barn are in relative motion, then from my point of view, the pole *really is* shorter than the barn.

A: No you can't really mean that. Just changing your point of view doesn't change what is really real! I mean, look at this pencil (holding up a pencil pointing towards Betty). To me it looks like a pencil but to you it looks like a tiny disc. But we both know that it really is a pencil even though it looks different from different angles.

B: Yes I do take your point – but there is something about the effects of special relativity which make them qualitatively different from other 'illusions' like the apparent bending of a stick placed in water or so-called 'fictitious forces' which appear to act on an object placed in a rotating frame of reference. Whether you call these effects 'real' or not is basically up to you but the thing which sets the effects of special relativity apart is that while in the other cases there is an obvious frame of reference which we can all agree on as being the ultimate 'reality' in SR all frames of reference are equally valid.

Obviously we do not want to say that there is no such thing as reality. But the only alternative is to say that all observers carry around a different reality with them.

A: So you are saying the the shortening of my pole is not an illusion – it is really real?

B: Yes – but you mustn't misunderstand me. All the measurements which I can make with my sophisticated clocks and measuring devices tell me that your pole *really is* 3 m long. But I am perfectly happy to accept that all the measurement *you* make on the pole tell you that it is actually 5m long. All I am saying is that it is perfectly OK for us to agree to disagree.