The Twins Paradox

Albert decides to travel in a fast spaceship at 80% of the speed of light to our nearest star Alpha Centauri which is 4 light years away leaving his twin brother Ludwig behind on Earth. Not knowing much about relativity he and his brother expect the journey to take 5 years there and 5 years back. (Time = distance / speed = 4 / 0.8 = 5 years). To his astonishment, Albert finds that the journey takes less time than he thought. 3 years, to be precise. Not finding anything of interest at Alpha Centauri, he heads for home. Arriving back at Earth only 6 years older (a fact verified by the observation that he had only eaten a little more than half the food he had taken with him) an even greater surprise awaits him. His brother Ludwig swears that he has been away 10 years after all - a fact amply proved by Ludwig's new wife and a family of 10 children!

The story is surprising enough but the fact that time proceeds more slowly for Albert than for his brother is not paradoxical. It is a straightforward consequence of Einstein's theory of Special Relativity; a consequence which has been verified a thousand times over by a great number of very different experiments to a remarkable degree of accuracy. However much you might find the result unpalatable, it remains a fact and contains no paradox.

So where *is* the real paradox? Let us continue the story. During Albert‘s absence, Ludwig had found a book about Relativity in the local library and had studied it closely. He had found it so interesting, in fact, that he had faxed a copy to his brother in the spaceship. (By a curious coincidence, he had faxed it at the precise moment when he had reckoned his brother would be arriving at Alpha Centauri - ie 5 years after his departure). In the book he had learned about time dilation and so he knew that Albert would only be 6 years older when he returned. When they were reunited, he was delighted to see his brother in such good health

Albert, on the other hand, was utterly dismayed when he saw his brother. Owing to the time it takes light (and radio waves) to travel, he had received his copy of the book on his way home and had not had time to read it properly. He had, however, got to the bit about time dilation and he had reasoned as follows: "All motion is relative, according to Einstein, so I might as well assume that I am the stationary one and that it is my brother Ludwig who is travelling at high speed away from me. Since he is in motion (relative to me) his clocks must be going slow (relative to mine) so when I get home I will find that *he* is younger than *me*." As events were to prove, he was mistaken - but what is wrong with his reasoning?

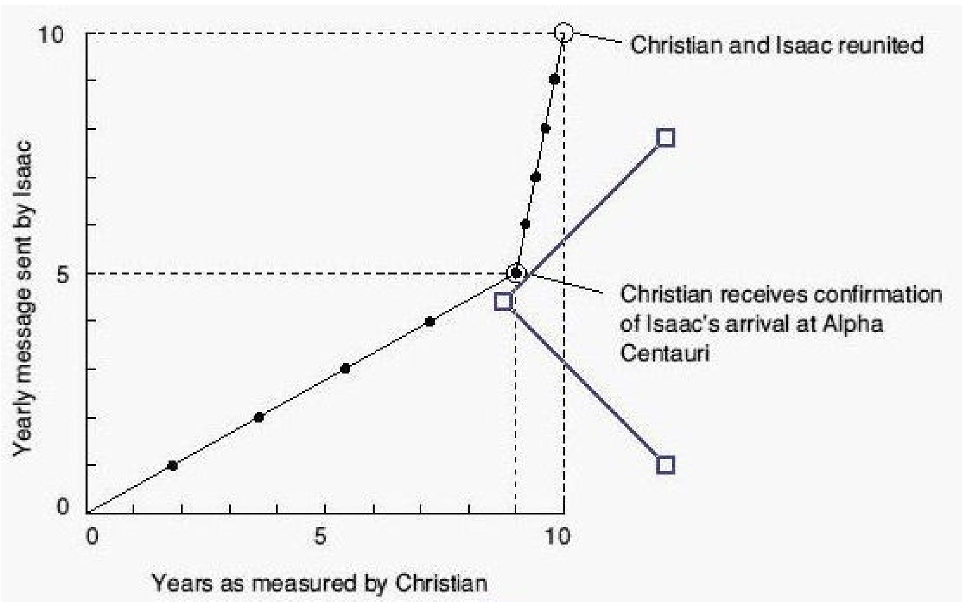
I have read several explanations of this paradox and I have never been satisfied with any of them. Some juggle with Lorentz transformations and prove it mathematically[2]. Others point to the fact that the situation is not, in fact, symmetrical because Albert undergoes accelerations and decelerations during which the principles of *Special* Relativity do not apply[3]. The first approach does not help if you are not a mathematician and the second is at best misleading and quite possibly wrong.

Certainly there is no need to invoke any new principles from Einstein's General theory to explain the paradox and while it is true that the situation is not symmetrical, the paradox arises precisely because there is an unexpected and pleasing symmetry in Einstein's solution which is absent from the non-relativistic expectations of the two brothers.

To explain what I mean, it is necessary to examine very carefully indeed exactly what the two brothers *expect* to happen during the journey assuming that there are no relativistic effects at all. To emphasise that we are discussing a classical, Newtonian world, we shall call our explorer Isaac and our stay-at-home brother, Christian.

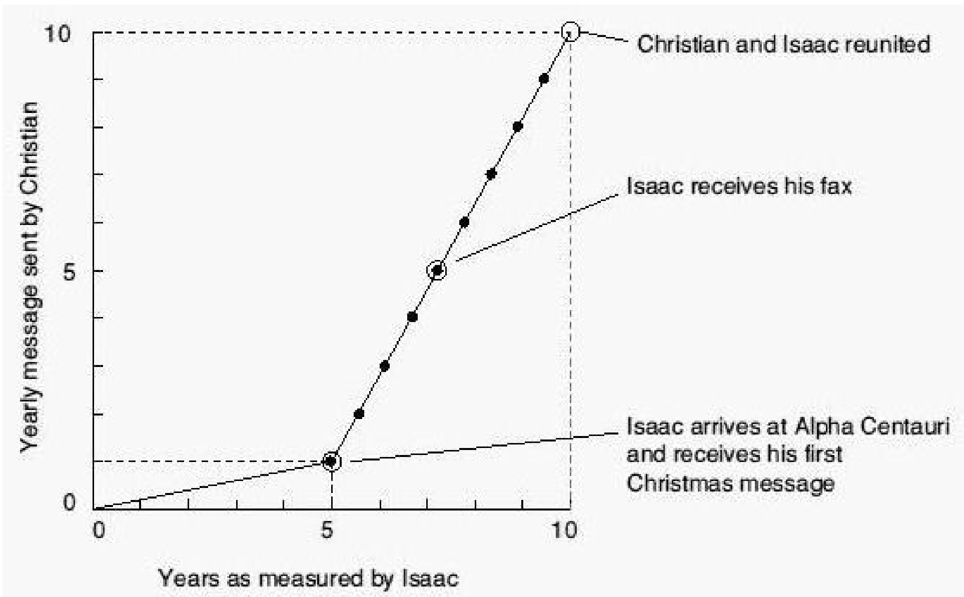
Let us suppose that Isaac sets out on Christmas Day on his 5 year voyage to Alpha Centauri. As a parting gift, Christian gives to Isaac a lovely clock which shows not only the time but the date and the year in big luminous letters. "With this clock," said Christian, "you will be able to tell exactly what time it is back home whenever you want." "Thank you so much! I have a present for you too." his brother replied, pulling out a small but beautifully made telescope. "You can watch me fly away with this and if I put my new clock in the window of the spaceship, you will be able to tell what time it is aboard ship whenever you want!". Both brothers were so delighted with their presents they decided to buy another clock and another telescope so that whenever they wanted to, they could look down their telescopes and see what their brother was doing and what time it was. They also agreed that throughout the voyage they would keep in contact by sending regular Christmas radio messages to each other.

Now both brothers were well aware that light (and radio waves) travel at a finite speed. They were quite prepared therefore to accept that, when looking through their telescopes, the brothers clock would not look as if it was saying the same time as their own; they also knew that they would receive their brother's messages long after they were sent. In order to keep track of things, each brother prepared a graph showing when (as shown on *his* clock) he would expect to receive his brothers messages. This is what the Earth-bound Christian's graph looks like.



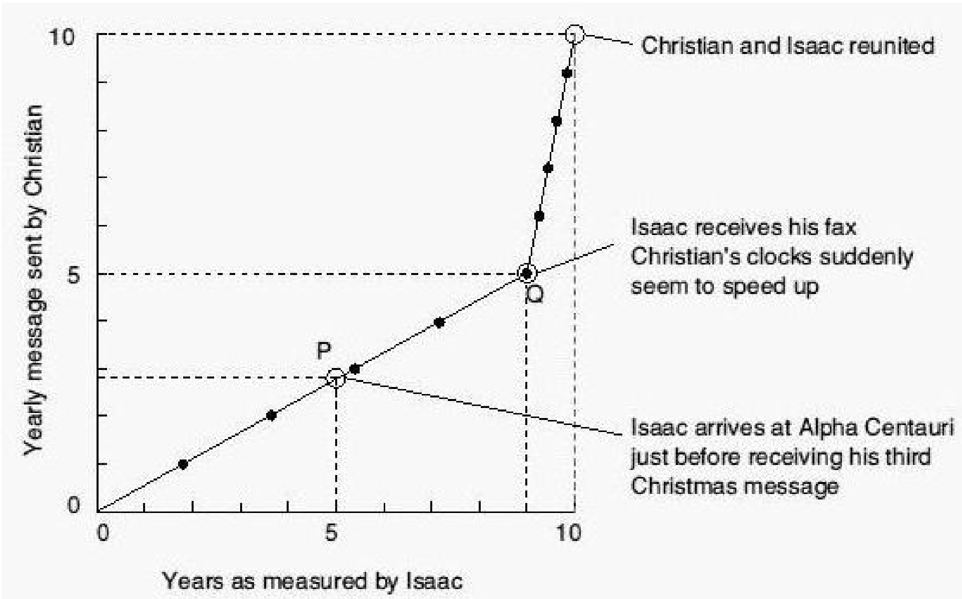
On the X axis we have Christian's time scale in years from 0 to 10. Since it will take 5 years for Isaac to reach Alpha Centauri and 4 years for the message to return home, Isaac's *fifth* Christmas message in which he confirms his arrival at his destination will arrive *nine* years after his departure. It follows that the first 5 messages will be received at 1.8, 3.6, 5.4, 7.2 and 9.0 years. This is shown on the graph by plotting a series of dots against the relevant year. The remaining messages come a lot closer together because Isaac is on his way home. In effect, the graph plots Isaac's time (as seen by Christian through a telescope) against Christian's time. (Remember - we are assuming that there are *no* relativistic effects, only effects due to the finite speed of light.)

Now what does Isaac's graph look like? The situation is a little more complicated here because Isaac can make one of two assumptions. The first is to accept that it is he who is moving through space and that when he is travelling away from Earth, messages take longer to reach him because the radio waves have to catch him up as he speeds away from the transmitter. (Once again, I must remind you, we are not talking *Relativity* here!), On the outward journey, Isaac is receding from Earth at 80% of the speed of light. One year out (when Christian sends his first message) he is 0.8 light years away. The radio waves are chasing him at the speed of light but the closing speed is only 0.2 light years per year. The radio waves will therefore take 0.8 / 0.2 = 4 years to catch him up and will therefore reach him at the precise instant that he reaches his destination. On the return leg, the closing speed between Isaac and the incoming messages is 1.8 times the speed of light. At this closing speed the time taken for a radio wave to meet the homecoming Isaac will be 1/1.8 = 0.55 years so Isaac will receive the remaining 9 messages in 9 * 0.55 = 5 years.



It will be noted that, although the two graphs are rather different, both brothers expect to be exactly 10 years older when they meet again! It is also worth noting that on the outward journey, both see their brothers clocks apparently running slow - but by different amounts. Christian receives 5 messages in 9 years, a time factor of 5/9 or 0.555. Isaac receives only one message in 5 years, a time factor of 0.2. (This difference is due to the fact that Christian experiences the Moving Source Effect while Isaac sees the Moving Observer Effect.) On the return leg, the time factors are 5 and 1.8 respectively.

But suppose that Isaac were to insist that it was *he* who was stationary and that it was Christian that was moving. Then, of course we would be back to the Moving Source Effect and Isaac would expect to see the same effects that we previously worked out for Christian. In this scenario you must imagine Isaac in his stationary rocket. For 5 years, Earth recedes and Alpha Centauri approaches, then for 5 years Earth approaches and Alpha Centauri recedes. The following graph shows when Isaac would receive his brother's messages.

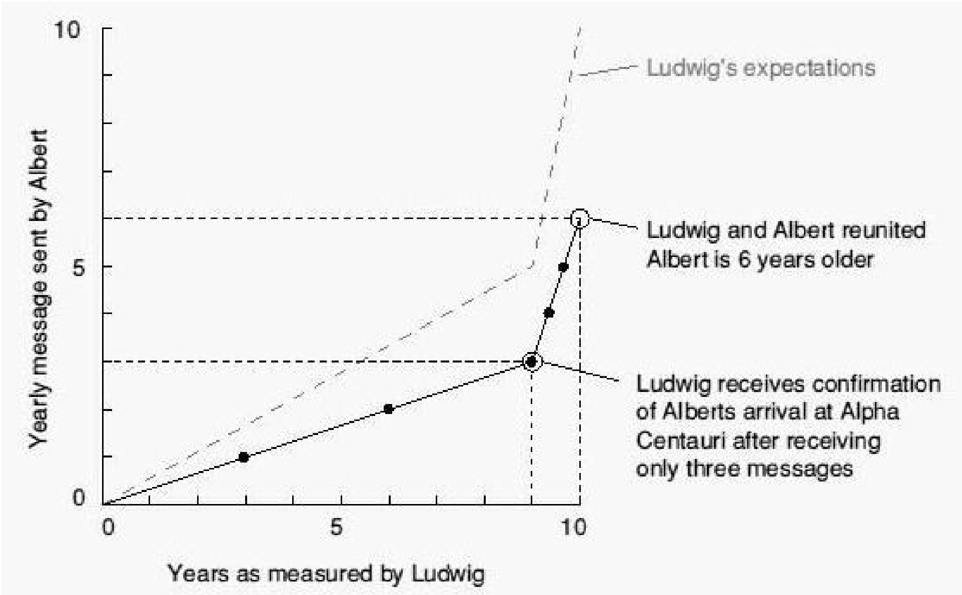


Note that under these circumstances, events turn out differently. Instead of receiving his first Christmas message when he arrives at his destination, Isaac will receive his third a short time later; and the changeover point when his brother's clocks change from going slow to going fast occurs not when he arrives at Alpha Centauri, but several years later, when he receives his fax.

It is, I hope, clear from this that in a non-relativistic world, events will turn out differently depending on who is stationary and who is moving (relative to the supposed medium in which light travels - the aether). The miracle of the Special Theory of Relativity is that it doesn't matter who is moving and who is stationary, the results always turn out to be the same - only the results aren't always what you expect! So how does Special Relativity achieve the miracle of allowing both Albert and Ludwig (the relativistic pair) to see their brother's clocks going slow, and yet allow one to be older than the other when they meet? The answer lies in contraction of length and the dilation (ie expansion) of time.

It is widely known that the special theory of relativity predicts that, as measured by any observer who considers himself stationary, moving clocks will run slow and that moving metre rulers will appear shorter by a factor which is the same in both cases and is equal to $\sqrt{1 - v^2/c^2}$. In the case we are considering the factor is equal to $\sqrt{1 - 0.8^2} = 0.6$. When Ludwig learned about time dilation, he realised that his astronaut brother Albert would only be 6 years older when he got home because for

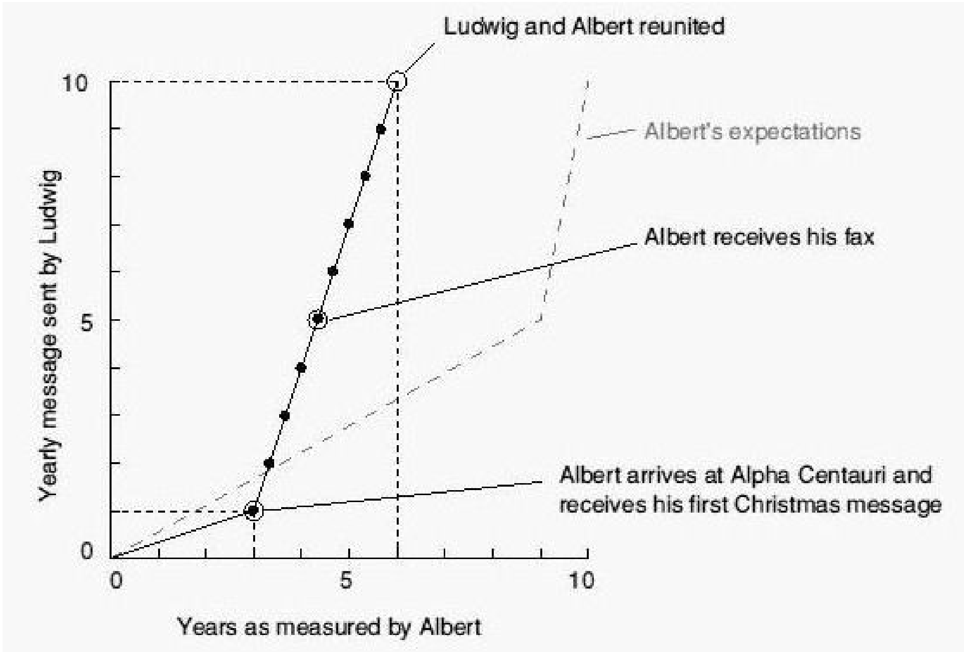
10 years, his brothers clocks would be going slower than his own. During the 10 year voyage, Ludwig therefore only receives 6 Christmas messages and, *compared to his expectations in the absence of Relativity*, Albert's clocks appear to be running slow by a factor of 0.6. This can be represented by the following graph. (The faint dashed line represents Ludwig's (non-relativistic) expectations for comparison)



The total time factor as measured by Ludwig on the outward journey is 3/9 or 0.33 and is the product of the doppler shift factor 5/9 and the time dilation factor 0.6.

Now why does Albert only send three messages before his arrival? The answer is that, to Albert, he is, by definition, stationary (*all* observers are by definition stationary in their own frame of reference!) and it is the Earth which really is receding and Alpha Centauri which really is approaching! To Albert, therefore, the Earth and Alpha Centauri constitute a huge cosmic ruler which was 4 light years long before he started moving. Here comes the crucial point. Because he is travelling so fast, *the length of this ruler shrinks* to 0.6 of 4 which is *2.4 light years* so it only takes 2.4 / 0.8 years to go by at a speed of 0.8 light years per year. This is, of course, exactly 3 years. The same is true on the way home. *That* is why Albert makes the round trip in 6 years. He doesn't have to go as far as he thought!

What about the messages that he receives from his brother? Because of the exact symmetry of the relativistic situation, *Albert must see exactly the same total doppler shift in Ludwig's messages as Ludwig sees in Albert's*. On the outward journey, Ludwig got 3 messages in 9 years. Albert must therefore receive exactly 1 message in the 3 years it takes him to reach Alpha Centauri. By the same token, he will receive 9 messages on the return journey. His graph therefore looks like this (as before, the faint dashed line represents Albert's (non-relativistic) expectations based on him being stationary and Ludwig receding from him):



We are now at long last, in a position to see clearly in what sense it is true for *both* brothers to claim that the other brother's clocks are running slow and yet for the two brothers to have different ages at the end of the voyage. For Ludwig, the effects of time dilation on his brother are immediately obvious. Albert is 4 years younger than he 'ought' to be! For Albert, the argument is a little more subtle. Once again, because of the exact symmetry of the relativistic doppler shift effect, Albert *expects* to see the same pattern of messages as his brother - ie 5 messages in the first 9 years and 5 in the last year. What he *actually* sees is 1 in the first 3 (a slower rate) and 9 in the next 3 (also a slower rate). In short, the *gradients* of both Ludwig's and Albert's graphs are identical. Both see the other as being both doppler shifted and time dilated. What is different is that for Ludwig the slow rate lasts for 9 years and the quick one for 1 year resulting in Albert being only 6 years older when he gets home. For Albert, the slow rate only lasts for 3 years and the quick rate for all of another 3 years resulting in Ludwig being 10 years older when they are reunited.

As they say in the ad - 'it's all worked out beautifully.'

Perhaps the most important thing to take home from all of this is that the resolution of the Twins Paradox has nothing at all to do with accelerations and decelerations, nor has it anything top do with General Relativity or the warping of spacetime; It is simply a result of the combined effects of the classical doppler shift, length contraction and time dilation. The asymmetry between the two twins arises simply because it is Albert who swops from one inertial frame of reference to a different one - not his stay-at-home brother.

References

1 Davies, P. *About Time.* Viking **1995** 0-670-84761-5

2 Feynman, R. *Lectures on Physics*, Addison-Wesley **1963**

3 Coleman, James A. *Relativity for the Layman*. Penguin **1959**