Bell's Inequality

Classical theory

When a beam of polarized photons impinges on an ideal polarizer, the component of the electric vector which is transmitted is $\cos(\theta)$ and the component which is absorbed is $\sin(\theta)$. The proportion of energy which is transmitted is $\cos^2(\theta)$ and that absorbed in $\sin^2(\theta)$. Of course, $\cos^2(\theta) + \sin^2(\theta) = 1$ so energy is conserved.

We can therefore say the when an individual photon impinges on a polarizer at an angle θ, the *probability* of it being transmitted in $\cos^2(\theta)$ and the *probability* that it will be absorbed is $\sin^2(\theta)$

Now consider the situation where pairs of identically polarized photons are directed (in opposite directions) to two polarizers A and B. Let us suppose that polarizer A is vertical and that polarizer B is inclined at an angle θ to the vertical. If we assume the principle of separability (ie that no influence can be transmitted from A to be or vice versa) then the probabilities are mutually exclusive and therefore the probability that both photons will be transmitted is the product of the probabilities that each will be transmitted. Hence, for a pair of photons polarized at an angle of α to the vertical, the probability that they will be transmitted through both polarizers is:

$$\cos^2(\alpha)\,.\,\cos^2(\alpha-\theta) \qquad (1)$$

If we experiment with a large number of photons with random polarization, the proportion of photons P which are transmitted through both polarizers will be:

$$P = \frac{1}{\pi}\int_0^{\pi} \cos^2(\alpha-\theta)\,.\,\cos^2(\alpha)\; d\alpha \qquad (2)$$

Now I can't integrate to save my life but I can program a computer and this integral works out to be as follows:

θ (deg)	0	10	20	30	40	50	60	70	80	90
P	0.374	0.367	0.345	0.312	0.271	0.228	0.187	0.154	0.133	0.125

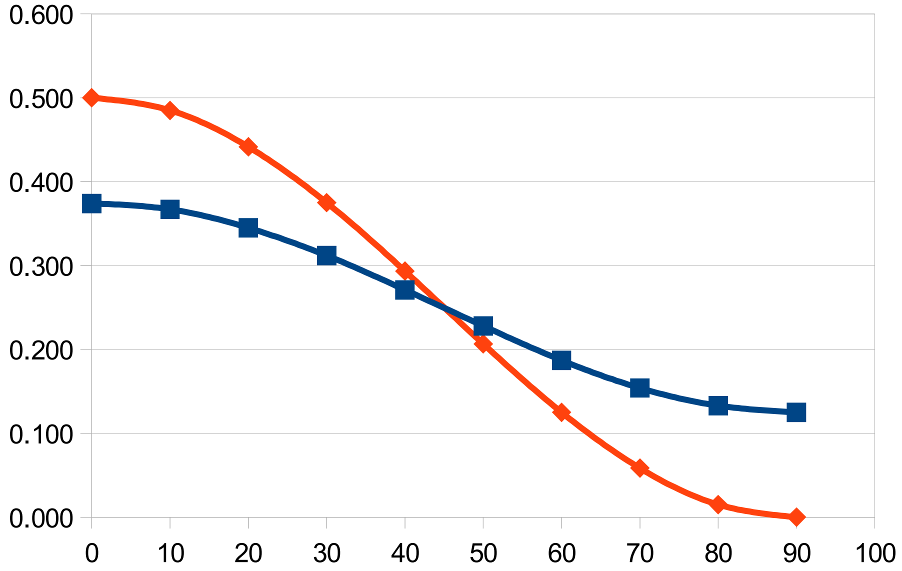
Quantum Theory

In the quantum world, the situation is different. Let us suppose that the vertical polarizer (A) is slightly closer to the source than polarizer B. The observation of the photon at A confirms that the photon reaching A is either vertically polarized or horizontally polarized – and that the other photon is therefore also similarly polarized. Under a conventional interpretation you can say that the observation of A collapses the wave function and creates a photon approaching B which is either vertically or horizontally polarized. Note carefully that the observation of the photon being either transmitted or absorbed at A *excludes* the possibility that the photon was polarized at any other angle.

Now, by symmetry, half the photons reaching A will be transmitted and half will be absorbed. For those photons which are transmitted, we know that its counterpart is vertically polarised and when it reaches polarizer B which is inclined at an angle θ, the probability that it will also be transmitted is, of course, $\cos^2(\theta)$. So out of a large number of photons, the proportion passing through both polarizers will be

$$P = 1/2\cos^2(\theta)$$

The differences are best shown on a graph with the classical prediction in blue and the quantum prediction in red:



Now it is not so much the fact that QT makes a quite different prediction; what is important is that no classical theory could possibly make the same prediction. (By classical theory, I mean one that ascribes fixed properties to both photons throughout the experiment.) It would be perfectly possible to dream up a theory of polarization, perhaps involving several 'hidden parameters', which would give the right answer at any articular angle between the two polarizers – but what Bell showed in his theorem was that *no* classical theory could ever give the right answers at *all* angles.