Tilings of the plane

Terminology

There appears to be some confusion as to the meanings of the terms semi-regular, demi-regular, quasi-regular etc. I shall use the following scheme instead.

Monomorphic – consisting of one shape of tile only
Bimorphic – consisting of two shapes of tile
Trimorphic - consisting of three shapes etc.

Regular – consisting of regular polygons only
Irregular – containing at least one irregular polygon

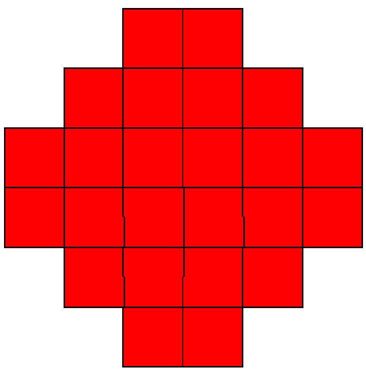
First order – containing vertices of one type only (ie all verticese are identical)
Second order – having two kinds of vertex
Third order – having three kinds of vertex etc.

Conformal – in which all the vertices coincide with vertices of another polygon
Non-conformal – in which vertices of one polygon lie on the edge of anotherRegular

Monomorphic Tilings

There are three conformal regular monomorphic tilings

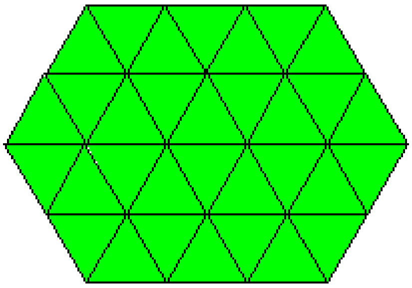
Square tiling



1S

It has 4-fold rotational symmetry, 4 reflection lines and 2 orthogonal translations
There are an infinite number of non-conformal variations with lesser symmetry.

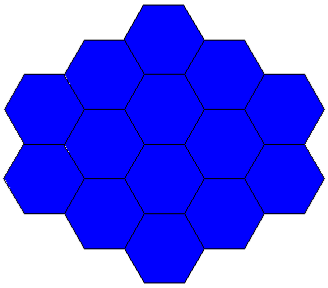
Triangular tiling



1T

It has 6-fold rotational symmetry, 3 reflection lines and 3 translations. Again, there are an infinite number of non-conformal variations with lesser symmetry

Hexagonal tiling



1H

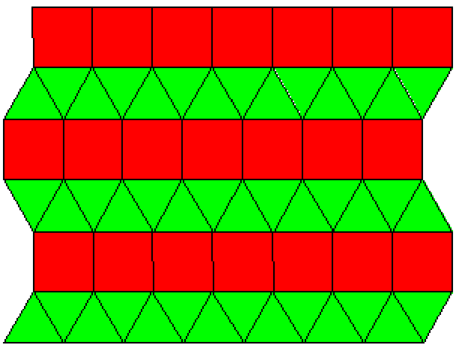
It has both 6-fold and 3-fold rotational symmetry

Regular Bimorphic Tilings

Squares and triangles

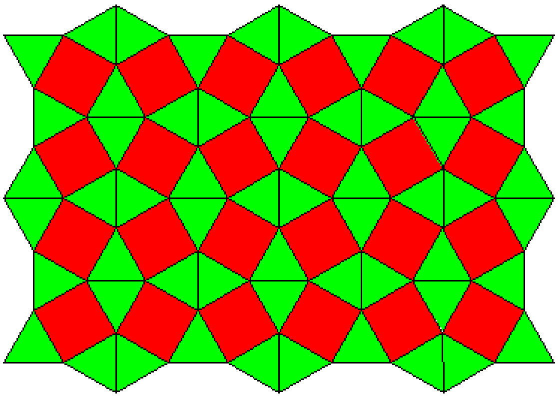
As far as I know there are only two first order regular bimorphic tilings containing squares and triangles

This is a first order regular tiling. Each vertex of this tiling has the order SSTTT.



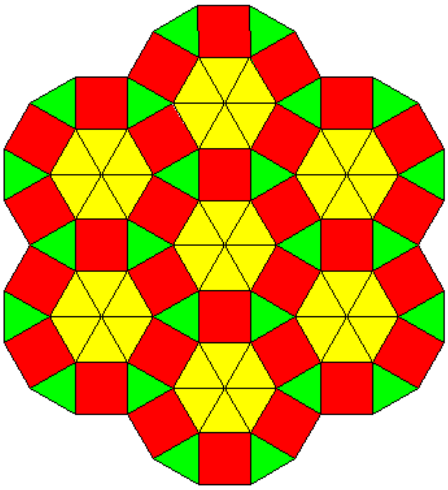
1S 2T

The order of this tiling is STSTT.



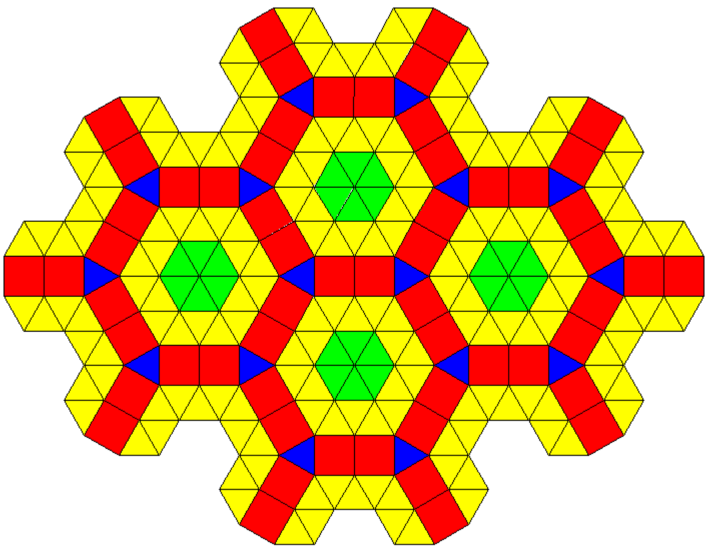
4S 8T

There appear to be a large number of regular bimorphic (ST) tilings of higher orders. Here is a selection. This one is of order 2. All the vertices of the squares are identical but the central vertex of the hexagon is different.



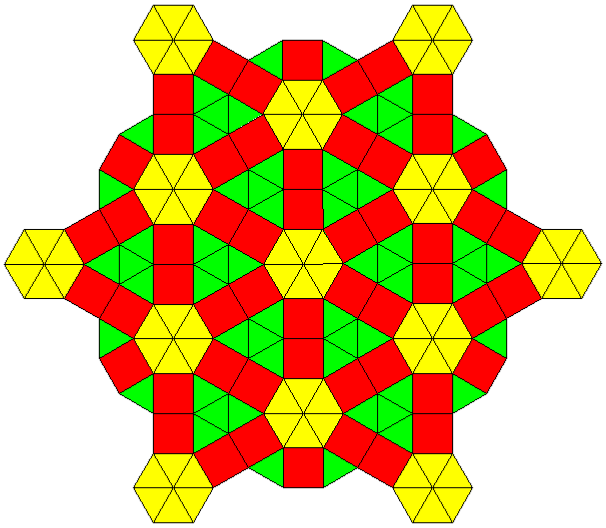
3S 8T

Here is another variation on the hexagonal theme. It is of order 4.
It is obvious that by increasing the size of the central hexagon and inserting spacers (the blue triangles), an infinite series of similar figures can be constructed



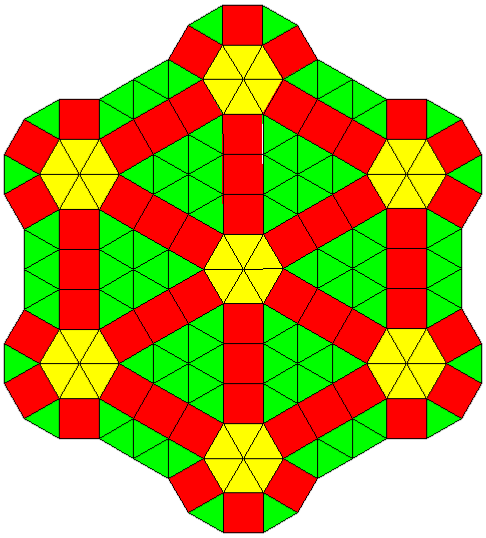
6S 26T

Alternatively, you can just lengthen the spokes as follows. Its order is 3.



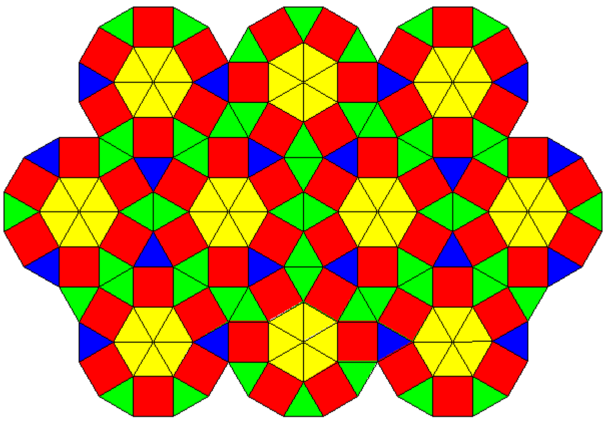
3S 7T

and so on..



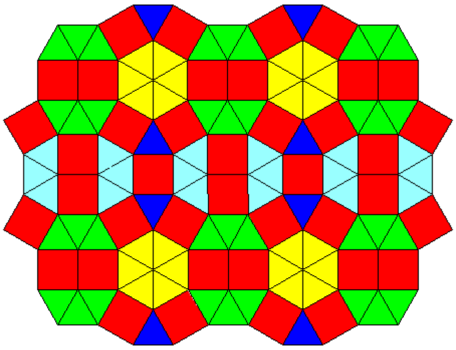
9S 24T

The next one is quite complex. Each vertex of the squares is different. Add the vertex at the centre of the hexagon and the order is 5. In spite of its hexagnonal appearance, the unit cell is rectangular.



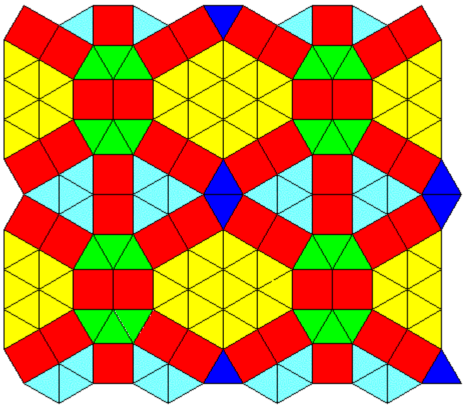
24S 56T

This one has rectangular symmetry. It has order 7.



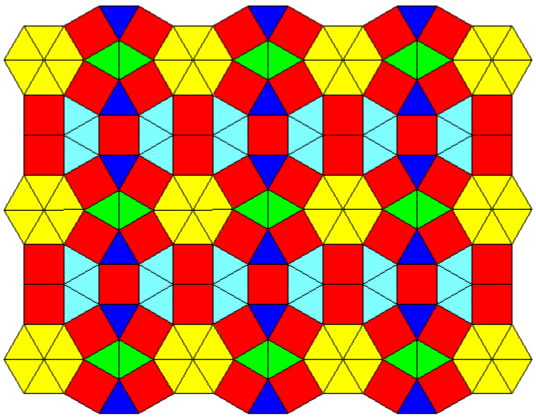
9S 20T

The next one has a very similar mode of construction.



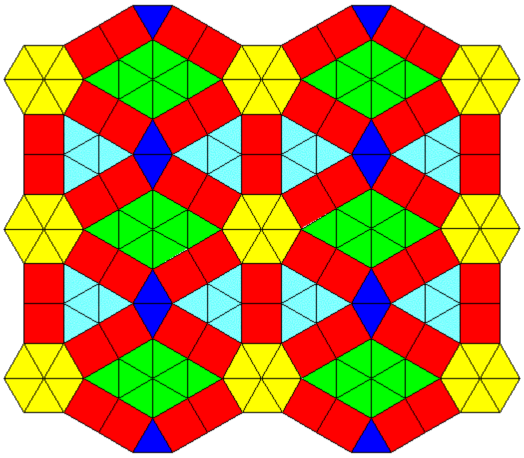
12S 30T

This one has a square lattice



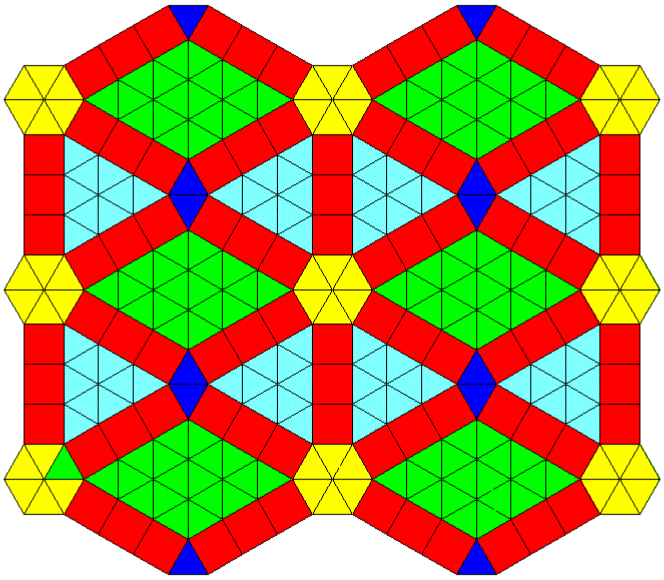
7S 16T

This one is similar but has a rectangular lattice



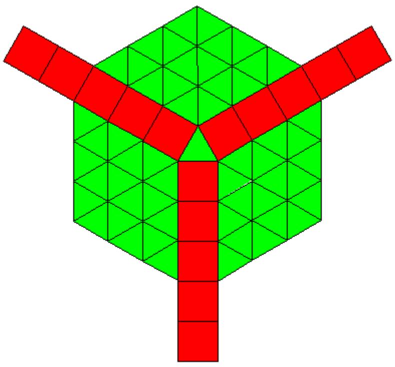
10S 24T

It is capable of being extended indefinitely.

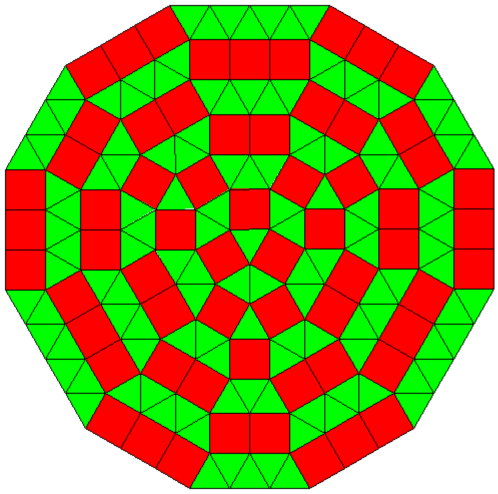


15S 44T

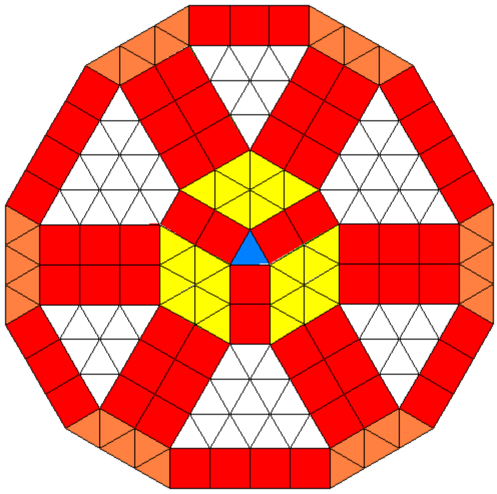
Not all tilings have translational symmetry. There is a whole class of tilings which only have rotational symmetry of which the most obvious is the propeller



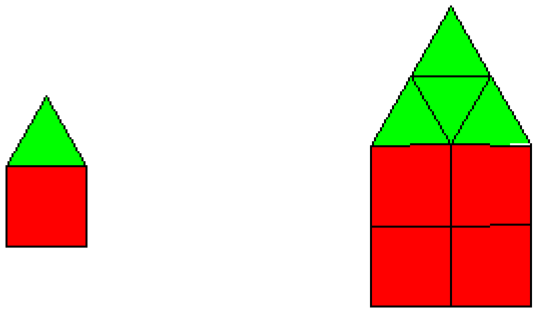
This is constructed in concentric rings by placing squares on squares and triangles on treiangles. If instead, we place triangles on squares and squares on triangles we get a spiders web:



It is possible to mix and match these two rules in any order you please giving an infinite variety of bizarre tilings eg:

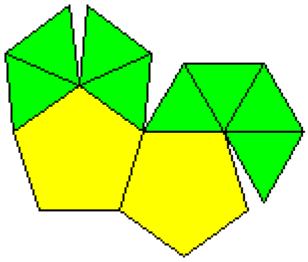


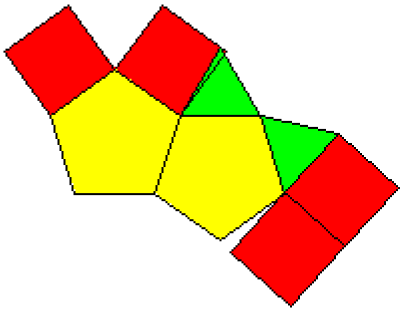
Any tiling can be doubled by replacing one square by four squares and one triangle by three triangles as shown below



Pentagons, Squares and Triangles

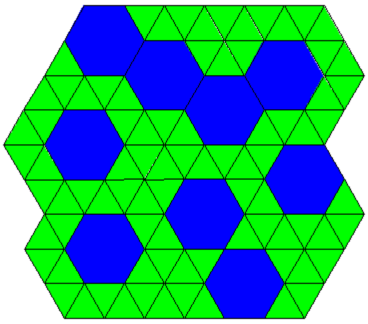
Pentagons are incompatible with either triangles or squares because the exterior angle of 252^0 cannot be filled with multiples of 60 and 90.





Hexagons, Squares and Triangles

Obviously all these tilings will reduce to squares and triangles because each hexagon can be freplaced by 6 triangles. There are an infinite number of bimorphic tilings with hexagons and triangles and it is easy to see that they do not have to have any symmetry at all as the following fragment illustrates:

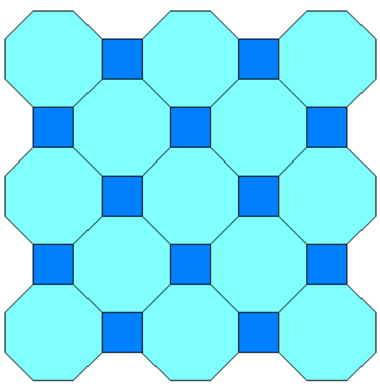


Heptagons

As with pentagons, there are no regular tilings with heptagons

Octagons

At last we find a new pattern here. This is the classic floor tiling. It is a first order tiling



There are no other bimorphic octagonal tilings